Inequalities

$a < b$	a is strictly less than b
$c > d$	c is strictly greater than d
$e \leq f$	e is less than or equal to f
$g \geq h$	g is greater than or equal to h

We found that adding or subtracting a constant from both sides of inequality did not invalidate the inequality.

Multiplying or dividing each side by a positive number did not invalidate the inequality

However, multiplying or dividing an inequality by a negative number caused the inequality to be reversed.

These observations suggest that inequalities can be solved much like equations.

Ex 1

Inequality	Equation
$2x + 3 < 11$	$2x + 3 = 11$
$2x < 11 - 3$	$2x = 11 - 3$
$2x < 8$	$2x = 8$
$x < \frac{8}{2}$	$x = \frac{8}{2}$
$x < 4$	$x = 4$

Ex 2

$$5x + 1 \geq 3x + 7$$
$$5x - 3x \geq +7 - 1$$
$$2x \geq 6$$
$$x \geq \frac{6}{2}$$
$$x \geq 3$$

$$5x + 1 = 3x + 7$$
$$5x - 3x = +7 - 1$$
$$2x = 6$$
$$x = \frac{6}{2}$$
$$x = 3$$

Ex 3

$$4x + 7 \leq 6x + 17$$
$$4x - 6x \leq +17 - 7$$
$$-2x \leq 10$$
$$x \geq \frac{10}{-2}$$
$$x \geq -5$$

$$4x + 7 = 6x + 17$$
$$4x - 6x = +17 - 7$$
$$-2x = 10$$
$$x = \frac{10}{-2}$$
$$x = -5$$

Ex 3 Alternative Solution

$$4x + 7 \leq 6x + 17$$
$$+7 - 17 \leq 6x - 4x$$
$$-10 \leq 2x$$
$$\frac{-10}{2} \leq x$$
$$-5 \leq x$$

$$4x + 7 = 6x + 17$$
$$+7 - 17 = 6x - 4x$$
$$-10 = 2x$$
$$\frac{-10}{2} = x$$
$$-5 = x$$

Exercise

1) $3x - 8 > 16$

$3x > 16 + 8$

$3x > 24$

$x > \frac{24}{3}$

$x > 8$

2) $5x + 1 \leq 16$

$5x \leq 16 - 1$

$5x \leq 15$

$x \leq \frac{15}{5}$

$x \leq 3$

3) $7x + 3 \geq 20$

$7x \geq 20 - 3$

$7x \geq 17$

$x \geq \frac{17}{7}$

$x \geq 2\,{}^{3}/_{7}$

4) $6x - 5 < 20$

$6x < 20 + 5$

$6x < 25$

$x < \frac{25}{6}$

$x < 4\frac{1}{6}$

5) $9x - 7 \leq 7x + 20$

$9x - 7x \leq 20 + 7$

$2x \leq 27$

$x \leq \frac{27}{2}$

$x \leq 13\frac{1}{2}$

6) $5x + 9 \geq 4 - 4x$

$5x + 4x \geq 4 - 9$

$9x \geq -5$

$x \geq -\frac{5}{9}$

$x \geq -\frac{5}{9}$

7) $3(x + 4) < 20$

$3x + 12 < 20$

$3x < 20 - 12$

$3x < 8$

8) $4(2x - 1) > 11$

$8x - 4 > 11$

$8x > 11 + 4$

$8x > 15$

$x < \frac{8}{3}$

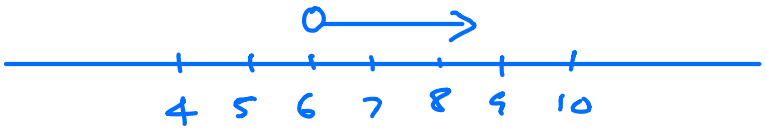
$x < 2\,2/3$

$x > \frac{15}{8}$

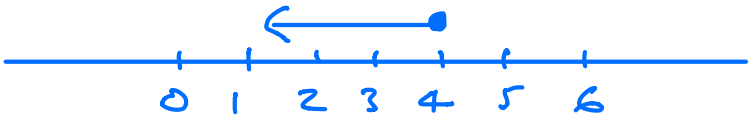
$x > 1\frac{7}{8}$

Representing Inequalities on the Number Line

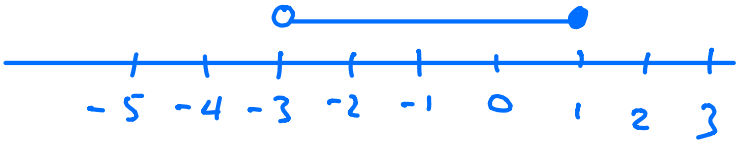
$x > 6$



$x \leq 4$



$-3 < x \leq 1$



Exercise

$x > 6$

$x \leq 4$

$-3 < x \leq 2$