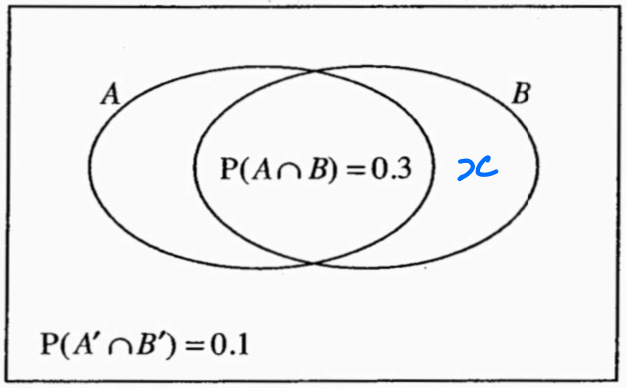
3 The Venn diagram illustrates the occurrence of two events A and B.

1



You are given that $\mathrm{P}(A \cap B) = 0.3$ and that the probability that neither A nor B occurs is 0.1. You are also given that $\mathrm{P}(A) = 2\mathrm{P}(B)$.

Find $\mathrm{P}(B)$. [3]

$$P(A \cup B) = 1 - 0.1 = 0.9$$

Let $P(B \cap A') = x$

Then $P(A \cap B') = 0.9 - 0.3 - x = 0.6 - x$

$$P(A) = 0.9 - x$$

$$P(B) = 0.3 + x$$

$$P(A) = 2P(B)$$

$$\Rightarrow \quad 0.9 - x = 2(0.3 + x)$$

$$0.9 - x = 0.6 + 2x$$

$$0.9 - 0.6 = 2x + x$$

$$0.3 = 3x$$

$$x = 0.1$$

$$\Rightarrow \quad \underline{P(B) = (0.3 + 0.1) = 0.4}$$

2 Isobel plays football for a local team. Sometimes her parents attend matches to watch her play.

- A is the event that Isobel's parents watch a match.
- B is the event that Isobel scores in a match.

2

You are given that $P(B \mid A) = \frac{3}{7}$ and $P(A) = \frac{7}{10}$.

(i) Calculate $P(A \cap B)$. [2]

The probability that Isobel does not score and her parents do not attend is 0.1.

(ii) Draw a Venn diagram showing the events A and B, and mark in the probability corresponding to each of the regions of your diagram. [2]

(iii) Are events A and B independent? Give a reason for your answer. [2]

(iv) By comparing $P(B \mid A)$ with $P(B)$, explain why Isobel should ask her parents not to attend. [2]

i)

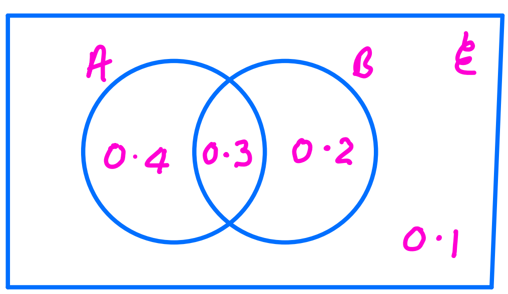
$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B \mid A) \times P(A)$$

$$= \frac{3}{7} \times \frac{7}{10}$$

$$= \frac{3}{10}$$

ii)



iii)

$P(A \cap B) = 0.3$ from diagram

$P(A) \times P(B) = 0.7 \times 0.5 = 0.35$

$\therefore P(A \cap B) \neq P(A) \times P(B)$ so A and B are not independent

iv) $P(B|A) = \frac{3}{7}$ $P(B) = 0\cdot5$

$0\cdot5 > \frac{3}{7}$ so Isabel has more chance of scoring when parents do not attend.

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5 Each day the probability that Ashwin wears a tie is 0.2. The probability that he wears a jacket is 0.4. If he wears a jacket, the probability that he wears a tie is 0.3.

(i) Find the probability that, on a randomly selected day, Ashwin wears a jacket and a tie. [2]

3

(ii) Draw a Venn diagram, using one circle for the event 'wears a jacket' and one circle for the event 'wears a tie'. Your diagram should include the probability for each region. [3]

(iii) Using your Venn diagram, or otherwise, find the probability that, on a randomly selected day, Ashwin

(A) wears either a jacket or a tie (or both),

(B) wears no tie or no jacket (or wears neither). [3]

i) Let A be event he wears a jacket
Let B be event he wears a tie

$P(A) = 0.4$ $P(B) = 0.2$ $P(B|A) = 0\cdot3$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B|A) \times P(A)$$
$$= 0.3 \times 0.4$$
$$= 0\cdot12$$

Prob he wears jacket and tie $= 0.12$

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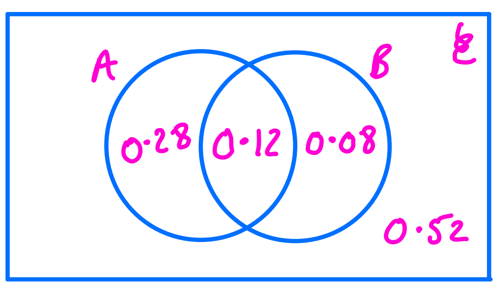
ii) A – Jacket B – Tie



iii)

A) $P(A \cup B) = 0.48$

B) $P(A \cap B)' = 0.88$

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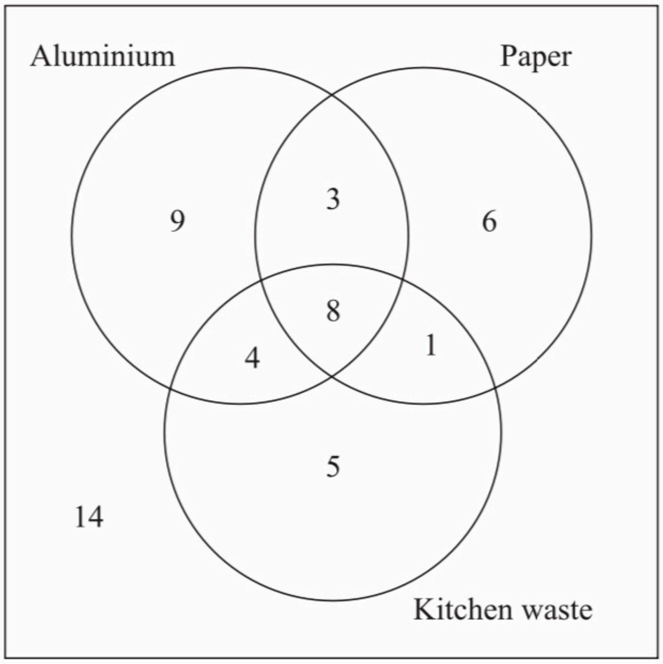
4 A local council has introduced a recycling scheme for aluminium, paper and kitchen waste. 50 residents are asked which of these materials they recycle. The numbers of people who recycle each type of material are shown in the Venn diagram.

4



One of the residents is selected at random.

(i) Find the probability that this resident recycles

(*A*) at least one of the materials, $\frac{36}{50}$ [1]

(*B*) exactly one of the materials. $\frac{9+6+5}{50} = \frac{2}{5}$ [2]

(ii) Given that the resident recycles aluminium, find the probability that this resident does not recycle paper. [2]

Two residents are selected at random.

(iii) Find the probability that exactly one of them recycles kitchen waste. [3]

ii) $\frac{13}{24}$

iii) $P(\text{Yes}, \text{No}) + P(\text{No}, \text{Yes})$

$$= \frac{18}{50} \times \frac{32}{49} + \frac{32}{50} \times \frac{18}{49}$$

$$= 0.4702$$

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2 In the 2001 census, people living in Wales were asked whether or not they could speak Welsh. A resident of Wales is selected at random.

- W is the event that this person speaks Welsh.
- C is the event that this person is a child.

5

You are given that $P(W) = 0.20$, $P(C) = 0.17$ and $P(W \cap C) = 0.06$.

(i) Determine whether the events W and C are independent. [2]

(ii) Draw a Venn diagram, showing the events W and C, and fill in the probability corresponding to each region of your diagram. [3]

(iii) Find $P(W|C)$. [2]

(iv) Given that $P(W|C') = 0.169$, use this information and your answer to part **(iii)** to comment very briefly on how the ability to speak Welsh differs between children and adults. [1]

i) $P(W) \times P(C) = 0.20 \times 0.17 = 0.034$

$P(W \cap C) = 0.06$

$0.06 \neq 0.034$

So W and C are not independent

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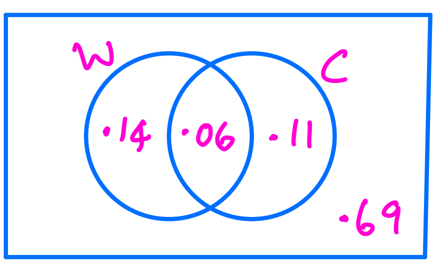
ii)



iii) $P(W|C) = \dfrac{P(W \cap C)}{P(C)} = \dfrac{0.06}{0.17} = \dfrac{6}{17}$

iv) $P(W|C) = \frac{6}{17} = 0.3529$

$P(W|C') = 0.169$

Children are just over twice as likely as adults to be able to speak Welsh.

3

6

5 Each day Anna drives to work.

- R is the event that it is raining.
- L is the event that Anna arrives at work late.

You are given that $P(R) = 0.36$, $P(L) = 0.25$ and $P(R \cap L) = 0.2$.

(i) Determine whether the events R and L are independent. **[2]**

(ii) Draw a Venn diagram showing the events R and L. Fill in the probability corresponding to each of the four regions of your diagram. **[3]**

(iii) Find $P(L \mid R)$. State what this probability represents. **[3]**

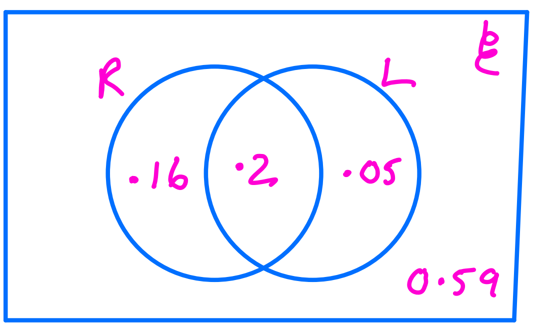
i) $P(R) \times P(L) = 0.36 \times 0.25 = 0.09$

$P(R \cap L) = 0.2$

$0.09 \neq 0.02$

so R and L are not independent

ii)



iii) $P(L|R)$ is the probability Anna arrives late given that it is raining

$P(L|R) = \frac{P(L \cap R)}{P(R)} = \frac{0.20}{0.36} = \frac{5}{9}$

7

3 In a survey, a large number of young people are asked about their exercise habits. One of these people is selected at random.

- G is the event that this person goes to the gym.
- R is the event that this person goes running.

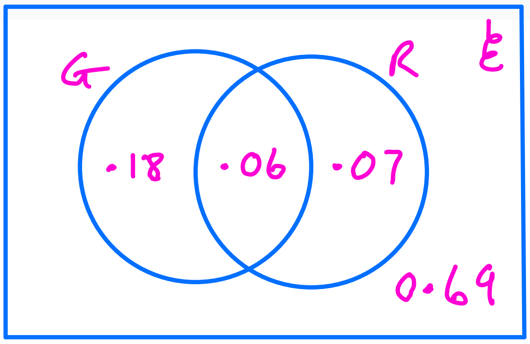
You are given that $P(G) = 0.24$, $P(R) = 0.13$ and $P(G \cap R) = 0.06$.

(i) Draw a Venn diagram, showing the events G and R, and fill in the probability corresponding to each of the four regions of your diagram. **[3]**

(ii) Determine whether the events G and R are independent. **[2]**

(iii) Find $P(R \mid G)$. **[3]**

i)



ii) $P(G) \times P(R) = 0.24 \times 0.13 = 0.0312$

$P(G \cap R) = 0.06$

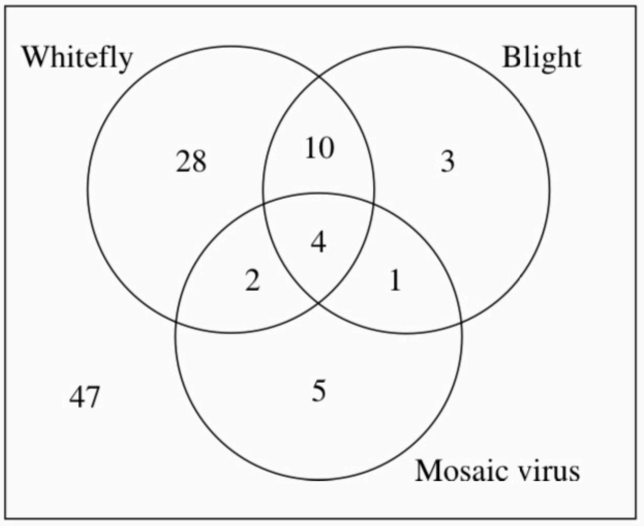
$0.06 \neq 0.0312$

$\therefore$ G and R are not independent

iii) $P(R \mid G) = \dfrac{P(R \cap G)}{P(G)} = \dfrac{0.06}{.24} = \dfrac{1}{4}$

8

6 Whitefly, blight and mosaic virus are three problems which can affect tomato plants. 100 tomato plants are examined for these problems. The numbers of plants with each type of problem are shown in the Venn diagram. 47 of the plants have none of the problems.



(i) One of the 100 plants is selected at random. Find the probability that this plant has

(A) at most one of the problems, **[1]**

(B) exactly two of the problems. **[2]**

(ii) Three of the 100 plants are selected at random. Find the probability that all of them have at least one of the problems. **[3]**

i) A) $\frac{83}{100}$ B) $\frac{10+2+1}{100} = \frac{13}{100}$

ii) $\frac{53}{100} \times \frac{52}{99} \times \frac{51}{98} = 0.1449$