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6. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

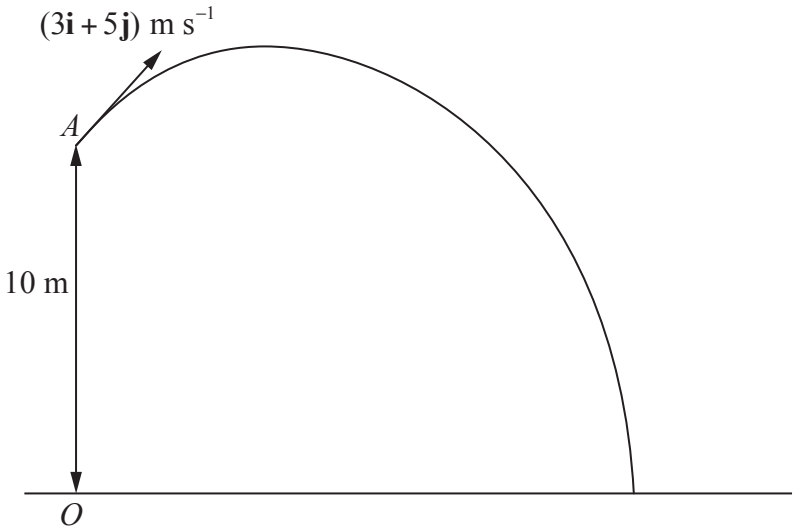


Figure 3

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i}+5\mathbf{j})$ m s^{-1}, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after T seconds.

(a) For $0 \leqslant t \leqslant T$, show that, with respect to O, the position vector, $\mathbf{r}$ metres, of P at time t seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$$

(3)

(b) Find the value of T.

(3)

(c) Find the velocity of P at time t seconds ($0 \leqslant t \leqslant T$).

(2)

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(d) Find the time taken for P to move from A to B.

(2)

(e) Find the speed of P as it passes through B.

(2)

a) ↔ $s_x = u_x t = 3t\,i$

↕ $s_y - s_0 = u_y t + \frac{1}{2} a t^2$

$s_y = (5t - 4.9t^2 + 10)\,j$

Leave blank

Question 6 continued

$$\underline{r} = 3t\,\underline{i} + (5t - 4.9t^2 + 10)\,\underline{j}$$

b) Hits ground when $t = T$

$$5T - 4.9T^2 + 10 = 0$$

$$4.9T^2 - 5T - 10 = 0$$

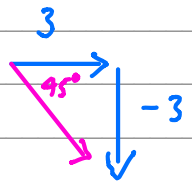
By calc $T = 2.03\text{ s}$, ~~-1.01 s~~

$$T = 2.03\text{ s}$$

c)

$$\underline{v} = \frac{d\underline{r}}{dt} = 3\,\underline{i} + (5 - 9.8t)\,\underline{j}$$

d)



Horizontal and Vertical speeds equal

$$5 - 9.8t = -3$$

$$-9.8t = -8$$

$$t = \frac{-8}{-9.8} = 0.816\text{ s}$$

Time from A to B $t = 0.82\text{ s}$

e)

Speed at B $= \sqrt{3^2 + (-3)^2}$

$= \sqrt{18}$

$= 4.24\text{ ms}^{-1}$

Leave blank

8. A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

(4)

A girl throws a ball from a point A at the top of a cliff. The point A is 8 m above a horizontal beach. The ball is projected with speed 7 m s^{-1} at an angle of elevation of 45°. By modelling the ball as a particle moving freely under gravity,

(b) find the horizontal distance of the ball from A when the ball is 1 m above the beach.

(5)

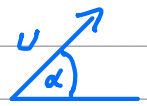
A boy is standing on the beach at the point B vertically below A. He starts to run in a straight line with speed v m s^{-1}, leaving B 0.4 seconds after the ball is thrown.

He catches the ball when it is 1 m above the beach.

(c) Find the value of v.

(4)

a)



$x = (u\cos\alpha)t$ (1)

$y = (u\sin\alpha)t - \frac{1}{2}gt^2$ (2)

From (1) $t = \frac{x}{u\cos\alpha}$

Sub for t in (2)

$y = \frac{(u\sin\alpha)x}{u\cos\alpha} - \frac{gx^2}{2u^2\cos^2\alpha}$

$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$

b)

$y = x\tan 45^\circ - \frac{gx^2}{2(7)^2\cos^2 45^\circ} + 8$

When 1m above ground $\quad 1 = x - \frac{9.8x^2}{98 \times \frac{1}{2}} + 8$

Leave blank

Question 8 continued

$$1 = x - \frac{x^2}{5} + 8$$

$$5 = 5x - x^2 + 40$$

$$x^2 - 5x - 35 = 0$$

By calc $x = 8.92\text{ m}$, -3.92 m

$$x = 8.92\text{ m}$$

c)

$$t = \frac{x}{u\cos\alpha} \qquad t = \frac{8.92}{7\cos 45^\circ}$$

$$t = 1.802$$

Boy runs for $1.802 - 0.4 = 1.402\text{ s}$

$$v = \frac{8.92}{1.402} = 6.36\text{ ms}^{-1}$$

$$v = 6.4\text{ ms}^{-1}$$

Leave blank

7. [*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively.*]

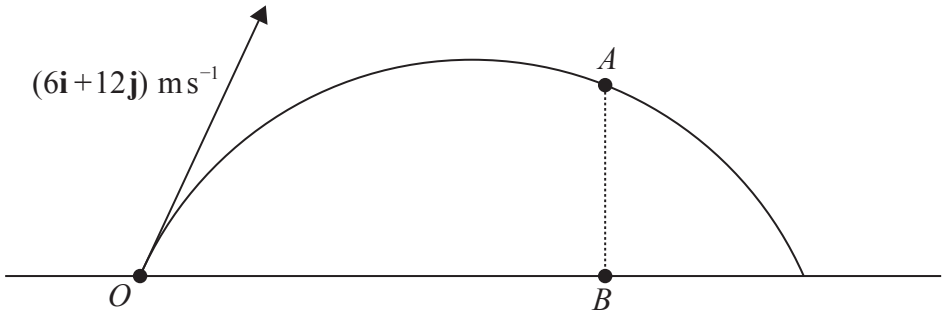


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

(3)

a) At time t $\quad x = 6t\,\underline{i}$

$y = (12t - 4.9t^2)\underline{j}$

Given $|x| = |2y|$

$\Rightarrow 6t = 2(12t - 4.9t^2)$

$6t = 24t - 9.8t^2$

$9.8t^2 - 18t = 0$

$t(9.8t - 18) = 0$

$t = 0$, $\quad t = \frac{18}{9.8} = 1.84\,s$

Leave blank

Question 7 continued

$t = 1.84 \text{ s}$

b)

$v_x = 6 \text{ ms}^{-1}$

$v_y = u_y + at = 12 - 9.8t$

$= 12 - 9.8 \times \frac{18}{9.8}$

$= -6 \text{ ms}^{-1}$

$\text{speed} = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{6^2 + (-6)^2}$

$= 8.49 \text{ ms}^{-1}$

c)

Same speed when $v_x = 6 \text{ ms}^{-1}$ $v_y = 6 \text{ ms}^{-1}$

$\Rightarrow 6 = 12 - 9.8t$

$9.8t = 12 - 6$

$t = \frac{6}{9.8}$

$t = 0.612 \text{ s}$

Leave blank

7.

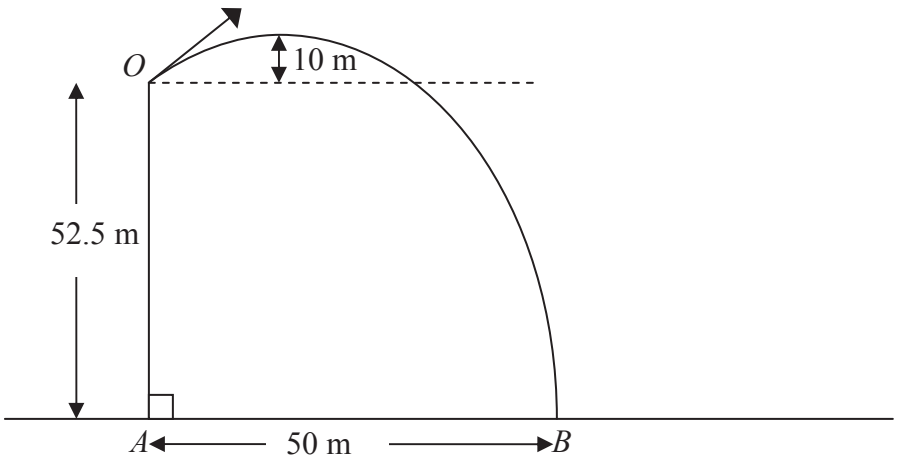


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.

(3)

(b) Find the speed of projection.

(9)

(c) Find the time after projection when the stone is moving parallel to OB.

(5)

a) $v_y^2 = u_y^2 + 2ay$

At top $v_y = 0$ $\quad 0 = u_y^2 - 2 \times 9.8 \times 10$

$196 = u_y^2$

$\sqrt{196} = u_y$

$u_y = 14 \text{ ms}^{-1}$

b) If time of flight = T

$-52.5 = 14T - 4.9T^2$

Leave blank

Question 7 continued

$4.9T^2 - 14T - 52.5 = 0$

By calc $T = 5\text{ s}$, ~~$-\frac{15}{7}$ s~~

Horizontally $x = u_x t$

$50 = u_x \times 5$

$\frac{50}{5} = u_x$

$u_x = 10\ \text{ms}^{-1}$

Speed of projection $= \sqrt{u_x^2 + u_y^2}$

$= \sqrt{10^2 + 14^2}$

$= 17.2\ \text{ms}^{-1}$

---

c) Moving parallel to OB when

$v_x : -v_y$

$= 50 : 52.5$

$= 20 : 21$

$\Rightarrow 21 v_x = -20 v_y$

$21 u_x = -20(u_y - 9.8t)$

$21 \times 10 = -20(14 - 9.8t)$

$\frac{210}{-20} = 14 - 9.8t$

$9.8t = 14 + 10.5$

$t = \frac{24.5}{9.8} = 2.5\text{ s}$

Leave blank

6.

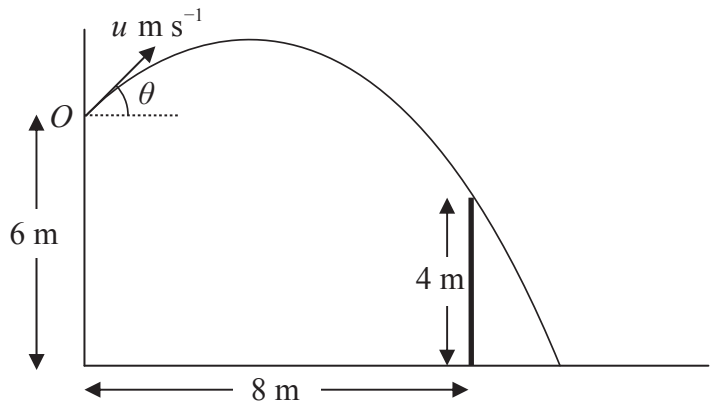


Figure 2

A ball is thrown from a point O, which is 6 m above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through O, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that $\tan\theta = 2.2$

(5)

(b) Find the value of u.

(2)

The ball hits the ground T seconds after projection.

(c) Find the value of T.

(3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(d) Find α.

(5)

a) $u_x = u\cos\theta$

Time to reach post $\dfrac{8}{u\cos\theta} = 2$ s

$8 = 2u\cos\theta$

$4 = u\cos\theta$ (1)

Leave blank

Question 6 continued

$y = u_y t - 4.9t^2$

At post $y = -2$, $t = 2$

$-2 = u\sin\theta \times 2 - 4.9 \times 2^2$

$-2 = 2u\sin\theta - 19.6$

$17.6 = 2u\sin\theta$

$8.8 = u\sin\theta$ (2)

(2) ÷ (1) $\quad \frac{u\sin\theta}{u\cos\theta} = \frac{8.8}{4}$

$\tan\theta = 2.2$

b)

$\theta = \tan^{-1} 2.2 = 65.556^\circ$

From (1) $\quad u = \frac{4}{\cos\theta} = \frac{4}{\cos 65.556^\circ}$

$u = 9.67\ ms^{-1}$

c)

Hits ground when $y = -6$

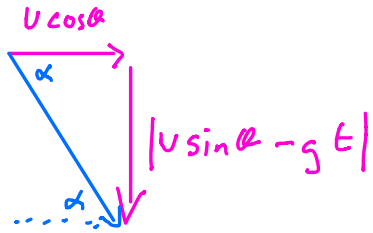
$-6 = 9.67\sin 65.556t - 4.9t^2$

$4.9t^2 - 9.67\sin 65.556t - 6 = 0$

By calc $\quad t = 2.32\ s \quad t = -0.527\ s$

$t = 2.32\ s$

d)



$$\tan\alpha = \frac{|U\sin\theta - g t|}{U\cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{|9.67\sin 65.556 - 9.8\times 2.32|}{9.67\times\cos 65.556}\right)$$

$$\alpha = 74.0^\circ$$

Leave blank

6.

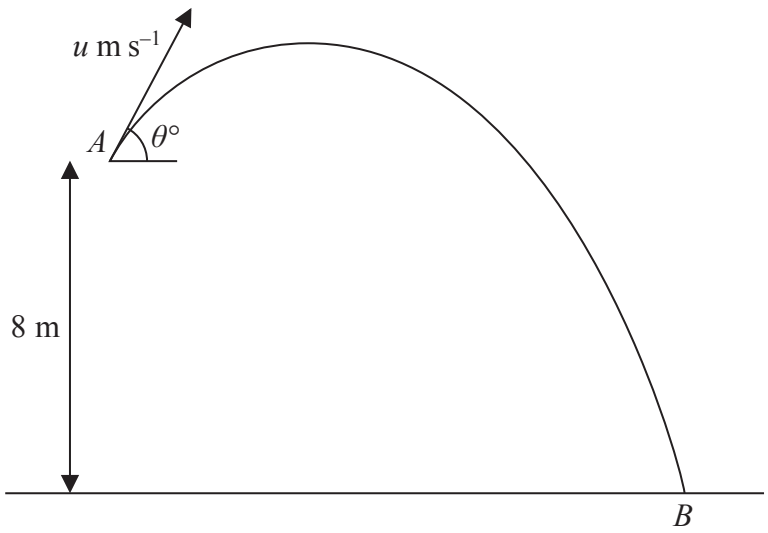


Figure 4

A ball is projected from a point A which is 8 m above horizontal ground as shown in Figure 4. The ball is projected with speed u m s^{-1} at an angle $\theta°$ above the horizontal. The ball moves freely under gravity and hits the ground at the point B. The speed of the ball immediately before it hits the ground is $2u$ m s^{-1}.

(a) By considering energy, find the value of u.

(5)

The time taken for the ball to move from A to B is 2 seconds. Find

(b) the value of θ,

(4)

(c) the minimum speed of the ball on its path from A to B.

(2)

The first part of this question is outside the new syllabus and requires the relationship: gain in kinetic energy = loss in gravitational potential energy

$0.5mv^2 - 0.5mu^2 = mgh$

a) $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgh$

$v^2 - u^2 = 2gh$

$(2u)^2 - u^2 = 2 \times 9.8 \times 8$

$3u^2 = 156.8$

Question 6 continued

Leave blank

$$U = \sqrt{\frac{156.8}{3}}$$

$$U = 7.23 \text{ ms}^{-1}$$

b) $y = U_y t - \frac{1}{2} g t^2$

$\begin{cases} y = -8 \\ t = 2 \end{cases}$ $\quad -8 = U\sin\theta \times 2 - 4.9 \times 2^2$

$$-8 = 14.46\sin\theta - 19.6$$

$$\frac{11.6}{14.46} = \sin\theta$$

$$\theta = 53.3^\circ$$

c) constant horizontal speed so minimum speed is when vertical speed = 0

Then speed $= U\cos\theta$

$= 7.23\cos 53.3^\circ$

$= 4.3 \text{ ms}^{-1}$