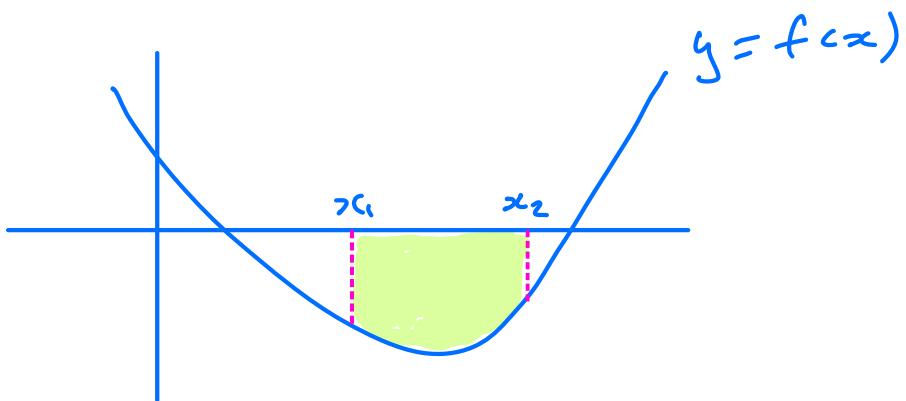


Area Under a Curve or Above a Curve

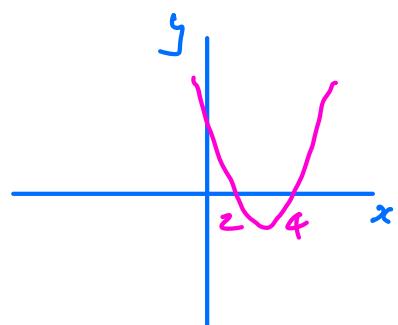


If area is below x -axis then $\int_{x_1}^{x_2} y dx$
will be negative, because the
the small rectangles have area $y \times \Delta x$ and $y < 0$

If asked to find the area shaded it
simply necessary to ignore the sign when the
integral has been found.

$$\text{Ex } f(x) = x^2 - 6x + 8 = (x-2)(x-4)$$

Find $\int_2^4 f(x) dx$



$$= \int_2^4 (x^2 - 6x + 8) dx$$

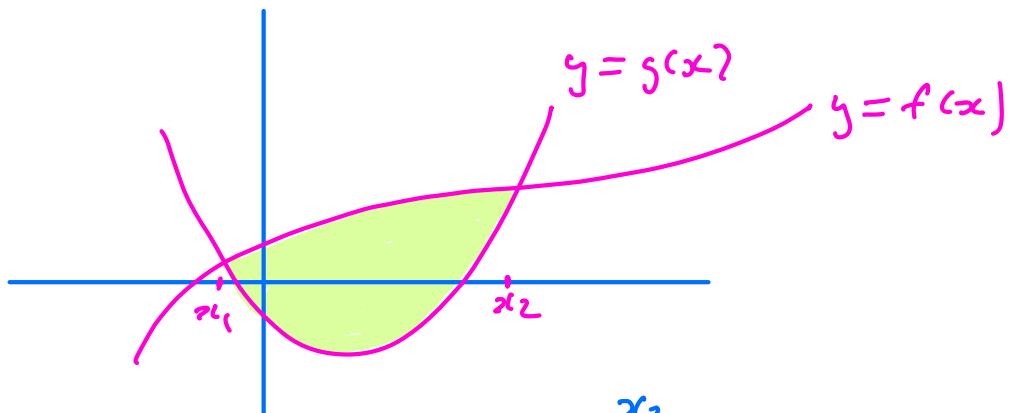
$$= \left[\frac{x^3}{3} - \frac{6x^2}{2} + 8x \right]_2^4$$

$$= \left[\frac{x^3}{3} - 3x^2 + 8x \right]_2^4$$

$$\begin{aligned}
 &= \left(\frac{4^3}{3} - 3(4)^2 + 8(4) \right) - \left(\frac{2^3}{3} - 3(2)^2 + 8(2) \right) \\
 &= \left(\frac{64}{3} - 48 + 32 \right) - \left(\frac{8}{3} - 12 + 16 \right) \\
 &= \frac{64}{3} - 16 - \left(\frac{8}{3} + 4 \right) \\
 &= \frac{56}{3} - 20 = \frac{56}{3} - \frac{60}{3} = -\frac{4}{3}
 \end{aligned}$$

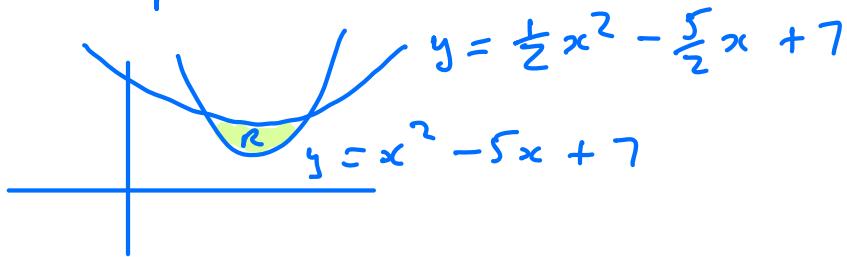
The area below a x -axis would be $\frac{4}{3}$

Area Between Two Curves



Area given by $\int_{x_1}^{x_2} (f(x) - g(x)) dx$

Example



Find area R

Find x coords of points of intersection

$$\frac{1}{2}x^2 - \frac{5}{2}x + 7 = x^2 - 5x + 7$$

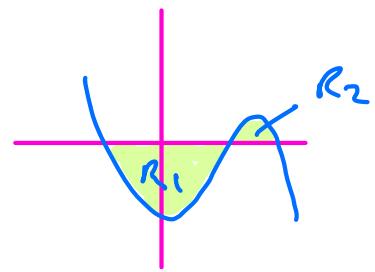
$$0 = \frac{1}{2}x^2 - \frac{5}{2}x$$

$$0 = \frac{1}{2}x(x-5)$$

$$\Rightarrow x = 0 \text{ or } x = 5$$

$$\begin{aligned} \text{Area } R &= \int_0^5 \left(\left(\frac{1}{2}x^2 - \frac{5}{2}x + 7 \right) - (x^2 - 5x + 7) \right) dx \\ &= \int_0^5 \left(\frac{1}{2}x^2 - \frac{5}{2}x + 7 - x^2 + 5x - 7 \right) dx \\ &= \int_0^5 \left(-\frac{1}{2}x^2 + \frac{5}{2}x \right) dx \\ &= \left[-\frac{1}{6}x^3 + \frac{5}{4}x^2 \right]_0^5 \\ &= \left(-\frac{1}{6}(5)^3 + \frac{5}{4}(5)^2 \right) - (0 + 0) \\ &= -\frac{125}{6} + \frac{125}{4} \\ &= \frac{125}{12} \quad \text{or } 10\frac{5}{12} \end{aligned}$$

$$f(x) = -x^3 + 4x^2 + 11x - 30$$



d) $f(-3) = -(-3)^3 + 4(-3)^2 + 11(-3) - 30$
 $= +27 + 36 - 33 - 30$
 $= 0$

By factor theorem $(x+3)$ is a factor

5)
$$\begin{array}{r} -x^2 + 7x - 10 \\ \hline x+3 \left| \begin{array}{r} -x^3 + 4x^2 + 11x - 30 \\ -x^3 - 3x^2 \\ \hline +7x^2 + 11x \\ +7x^2 + 21x \\ \hline -10x - 30 \\ -10x - 30 \\ \hline \end{array} \right. \end{array}$$

$$f(x) = (x+3)(-x^2 + 7x - 10)$$

c) $f(x) = -(x+3)(x^2 - 7x + 10)$
 $f(x) = -(x+3)(x-2)(x-5)$

d) Intersects x -axis at
 $x = -3, x = 2, x = 5$

$$e) \quad R_1 = - \int_{-3}^2 y \, dx \quad R_2 = + \int_2^5 y \, dx$$

$$\begin{aligned}
R_1 &= - \left[-\frac{x^4}{4} + \frac{4x^3}{3} + \frac{11x^2}{2} - 30x \right]_{-3}^2 \\
&= - \left[\left(-\frac{2^4}{4} + \frac{4(2)^3}{3} + \frac{11(2)^2}{2} - 30(2) \right) \right. \\
&\quad \left. - \left(-\frac{(-3)^4}{4} + \frac{4(-3)^3}{3} + \frac{11(-3)^2}{2} - 30(-3) \right) \right] \\
&= - \left[\left(-4 + \frac{32}{3} + 22 - 60 \right) - \left(-\frac{81}{4} - 36 + \frac{99}{2} + 90 \right) \right] \\
&= - \left[-\frac{94}{3} - \frac{333}{4} \right] \\
&= \frac{1375}{12}
\end{aligned}$$

$$\begin{aligned}
R_2 &= \left[-\frac{x^4}{4} + \frac{4x^3}{3} + \frac{11x^2}{2} - 30x \right]_2^5 \\
&= \left(-\frac{5^4}{4} + \frac{4(5)^3}{3} + \frac{11(5)^2}{2} - 30(5) \right) - \left(-\frac{94}{3} \right) \\
&= -\frac{625}{4} + \frac{500}{3} + \frac{275}{2} - 150 + \frac{94}{3} = \frac{117}{4}
\end{aligned}$$

$$\text{Total Area} = R_1 + R_2 = \frac{1375}{12} + \frac{117}{4} = \frac{863}{6} = 143\frac{5}{6}$$