Surds

Consider $(\sqrt{a} \times \sqrt{b})(\sqrt{a} \times \sqrt{b})$

$$= \sqrt{a} \times \sqrt{b} \times \sqrt{a} \times \sqrt{b}$$

$$= \sqrt{a} \times \sqrt{a} \times \sqrt{b} \times \sqrt{b}$$

$$= a \times b$$

$$= ab$$

$$\therefore \quad \underline{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$$

Now consider $\dfrac{\sqrt{a}}{\sqrt{b}} \times \dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{\sqrt{a} \times \sqrt{a}}{\sqrt{b} \times \sqrt{b}} = \dfrac{a}{b}$

$$\therefore \quad \underline{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$$

These are the two main rules for multiplying and dividing surds

Examples of surds

$\sqrt{2}$, $\sqrt{3}$, $5+\sqrt{7}$, $\dfrac{3}{\sqrt{8}}$, $\dfrac{6+\sqrt{3}}{2-\sqrt{5}}$

There are such rules for addition and subtraction of surds

Counter Examples

$$\sqrt{9} + \sqrt{4} \neq \sqrt{9+4}$$

$$3 + 2 \neq \sqrt{13} \approx 3.60555$$

$$5 \neq 3.60555$$

More generally

$$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$$

$$\sqrt{9} - \sqrt{4} \neq \sqrt{9-4}$$

$$3 - 2 \neq \sqrt{5}$$

$$1 \neq 2.236$$

More generally

$$\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$$

Surds - 3 Main Tasks

Task 1

Ex1 Simplify $\sqrt{50} + \sqrt{18}$

$$= \sqrt{25 \times 2} + \sqrt{9 \times 2}$$

$$= \sqrt{25} \times \sqrt{2} + \sqrt{9} \times \sqrt{2}$$

$$= 5\sqrt{2} + 3\sqrt{2}$$

$$= 8\sqrt{2}$$

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Ex2 $\sqrt{27} + \sqrt{12} - \sqrt{75}$

$\sqrt{9\times3} + \sqrt{4\times3} - \sqrt{25\times3}$

$= 3\sqrt{3} + 2\sqrt{3} - 5\sqrt{3}$

$= 0$

Exercise Simplify

1) $\sqrt{8} + \sqrt{72} = \sqrt{4\times2} + \sqrt{36\times2} = 2\sqrt{2} + 6\sqrt{2} = 8\sqrt{2}$

2) $\sqrt{20} + \sqrt{45} = \sqrt{4\times5} + \sqrt{9\times5} = 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$

3) $\sqrt{162} - \sqrt{98} = \sqrt{81\times2} + \sqrt{49\times2} = 9\sqrt{2} - 7\sqrt{2} = 2\sqrt{2}$

4) $\sqrt{48} - 2\sqrt{3} = \sqrt{16\times3} - 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3}$

5) $3\sqrt{75} + 2\sqrt{12} = 3\sqrt{25\times3} + 2\sqrt{4\times3} = 15\sqrt{3} + 4\sqrt{3} = 19\sqrt{3}$

Ex3 $3\sqrt{28} + 5\sqrt{63}$

$= 3\sqrt{4\times7} + 5\sqrt{9\times7}$

$= 6\sqrt{7} + 15\sqrt{7}$

$= 21\sqrt{7}$

Task 2

Ex1 $(6+\sqrt{3})(5+\sqrt{3})$

$30 + 5\sqrt{3} + 6\sqrt{3} + 3 = 33 + 11\sqrt{3}$

Ex2 $(5+3\sqrt{2})(2+4\sqrt{2})$

$= 10 + 6\sqrt{2} + 20\sqrt{2} + 24$

$= 34 + 26\sqrt{2}$

Note $3\sqrt{2} \times 4\sqrt{2} = 3 \times 4 \times \sqrt{2} \times \sqrt{2}$

$= 12 \times 2$

$= 24$

Ex3 $(2+\sqrt{5})(3-2\sqrt{5})$

$= 6 + 3\sqrt{5} - 4\sqrt{5} - 10$

$= -4 - \sqrt{5}$

Ex4 $(2+\sqrt{3})(5+\sqrt{2})$

$= 10 + 5\sqrt{3} + 2\sqrt{2} + \sqrt{6}$

Exercise Expand and Simplify

1) $(4+\sqrt{3})(2+\sqrt{3}) = 8 + 2\sqrt{3} + 4\sqrt{3} + 3$

$= 11 + 6\sqrt{3}$

2) $(5+\sqrt{7})(3-\sqrt{7}) = 15 + 3\sqrt{7} - 5\sqrt{7} - 7$

$= 8 - 2\sqrt{7}$

3) $(2+3\sqrt{2})(5+\sqrt{2}) = 10 + 15\sqrt{2} + 2\sqrt{2} + 6$

$= 16 + 17\sqrt{2}$

4) $(7-\sqrt{5})(3-\sqrt{5}) = 21 - 3\sqrt{5} - 7\sqrt{5} + 5$

$= 26 - 10\sqrt{5}$

5) $(4-3\sqrt{2})(3-5\sqrt{2}) = 12 - 9\sqrt{2} - 20\sqrt{2} + 30$

$= 42 - 29\sqrt{2}$
