Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2E: Further Statistics 1 and Further Mechanics 1

Solutions

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference **8FM0/2E**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language			
		French	**Spanish**	**Mandarin**	
Gender	**Male**	23	22	20	65
	Female	38	32	15	85
		61	54	35	150

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

a) H_0: There is no association between gender and choice of language

b) Expected Spanish Females

$$= \frac{54 \times 85}{150}$$

$$= 30.6$$

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Question 1 continued

c) Expected frequencies in pink

		Language			
		French	Spanish	Mandarin	
Gender	Male	23 26.4	22 23.4	20 15.2	65
	Female	38 34.6	32 30.6	15 19.8	85
		61	54	35	150

$$X^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} = \sum_i \frac{O_i^2}{E_i} - N$$

$$= \frac{23^2}{26.4} + \frac{38^2}{34.6} + \frac{22^2}{23.4} + \frac{32^2}{30.6} + \frac{20^2}{15.2} + \frac{15^2}{19.8} - 150$$

$$X^2 = 3.599$$

d) $\nu = (3-1) \times (2-1) = 2$

Critical value for 1% sig level

$= 9.210$

$3.599 < 9.210$

$\therefore$ accept H_0 the null hypothesis

e) At 10% sig level critical value $= 4.605$

$3.599 < 4.605$

so null hypothesis would still be accepted

2. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $E(Y^2) = 75$

(b) find the exact value of $Var(X)$

(2)

(c) Find $P(Y > X)$

(2)

a)

y	7	2	-3	-8	-13
x	-1	0	1	2	3
$P(X = x)$	c	a	a	b	c

$E(Y) = 7c + 2a - 3a - 8b - 13c$

$-4 = -a - 8b - 6c$

$\Rightarrow a + 8b + 6c = 4$ (1)

$P(Y \geq -3) = a + a + c$

$\Rightarrow 2a + c = 0.45$ (2)

Also $c + a + a + b + c = 1$

$2a + b + 2c = 1$ (3)

Question 2 continued

Solve ①, ②, ③ simultaneously by calc

$a = \frac{1}{10}$ $b = \frac{3}{10}$ $c = \frac{1}{4}$

x	-1	0	1	2	3
$P(X = x)$	c	a	a	b	c

$P(X=x)$ 0.25 0.1 0.1 0.3 0.25

b) $Var(y) = E(y^2) - (E(y))^2$

$= 75 - (-4)^2$

$= 59$

$sd_y = 5\, sd_x$

$Var(y) = 25\, Var(x)$

$\therefore Var(x) = \frac{59}{25} = 2.36$

c) $P(y > x) = a + c$

$= 0.1 + 0.25$

$= 0.35$

3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

a) $A \sim Po(2.6)$ $B \sim Po(1.2)$

$P(A = 2 \cap B = 2)$

$= e^{-2.6} \times \frac{2.6^2}{2!} \times e^{-1.2} \times \frac{1.2^2}{2!}$

$= 0.0544$

b) $X \sim Po(2.6 + 1.2)$

$X \sim Po(3.8)$

Question 3 continued

$$P(X=3) = e^{-3.8} \times \frac{3.8^3}{3!}$$

$$= 0.2046$$

c) $Y \sim Po(7.6)$

$$P(Y<9) = P(Y \leq 8)$$

$$= 0.6482$$

d) $X \sim B(\overset{n}{600}, \overset{p}{0.004})$

i) $E(X) = np = 2.4$

ii) $Var(X) = npq = 2.4 \times 0.996 = 2.3904$

e) i) Approx with $Y \sim Po(2.4)$

$$P(Y \leq 4) = 0.9041$$

ii) n large and p is small with mean approximately equal to variance

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

a)

$X \sim Po(1.4)$

i) $P(X=1) = e^{-1.4} \times \frac{1.4}{1} = 0.3452$

ii) $P(X \leq 4) = 0.9857$

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Question 4 continued

b)

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

$$\text{Mean} = \frac{10\times0 + 16\times1 + 7\times2 + 4\times3 + 2\times4 + 0\times5 + 1\times6}{40}$$

$$= \frac{56}{40} = 1.4$$

c)

$$r = 40 \times P(X = 1)$$

$$r = 40 \times 0.3452$$

$$r = 13.81$$

$$s = 40 - (9.86 + 13.81 + 9.67 + 4.51 + 1.58)$$

$$s = 0.57$$

d)

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	13.81	9.67	4.51	1.58	0.57

group

6.66

Group last 3 so no expected values less than 5

$$X^2 = \sum_i \frac{O_i^2}{E_i} - N$$

$$X^2 = \frac{10^2}{9.86} + \frac{16^2}{13.81} + \frac{7^2}{9.67} + \frac{7^2}{6.66} - 40 = 1.104$$

Question 4 continued

H_0 : Poisson(1.4) is a suitable model for data

H_1 : Poisson(1.4) is not a suitable model for data

Constraints 1. $\Sigma O_i = \Sigma E_i$

2. λ estimated from data

$\therefore \quad \nu = 4 - 1 - 1 = 2$

Critical Value for 5% sig level $\chi^2_2(0.05) = 5.991$

$1.104 < 5.991$

Accept H_0

Poisson(1.4) is a suitable model for data

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SECTION B

Answer ALL questions. Write your answers in the spaces provided

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

5. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

a) $v^2 = u^2 + 2as$

$v^2 = 0 + 19.6 \times 2.4$

$v^2 = 47.04$

$v = \frac{14\sqrt{6}}{5}\ \text{m s}^{-1}$ = speed hitting floor

$v^2 = u^2 + 2as$

$0 = u^2 - 19.6 \times 0.6$

$u^2 = 11.76$

$u = \frac{7\sqrt{6}}{5}\ \text{m s}^{-1}$ = speed leaving floor

$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{7\sqrt{6}}{5} \div \frac{14\sqrt{6}}{5} = 0.5$

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Question 5 continued

b) $s = \frac{u^2}{2g}$

2nd ↑

$s = \frac{\left(0.5 \times \frac{7\sqrt{6}}{5}\right)^2}{2 \times 9.8} = 0.15\text{m}$

Reaches height of 0.15m after second bounce

c) Ever decreasing height bounces, each bounce $\frac{1}{4}$ of the height of the previous bounce

(Total for Question 5 is 10 marks)

6. A small stone of mass 0.5 kg is thrown vertically upwards from a point A with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point B which is 20 m vertically above the point A. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from A to B

(3)

The air resistance is modelled as a constant force of magnitude R newtons.

(b) Find the value of R.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

a)

Loss in KE = Gain in GPE + Work Against Air Resistance

$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh$ + Work Against Air Resistance

Energy lost to air resistance $= \frac{1}{2} \times 0.5 \times 25^2 - 0.5 \times 9.8 \times 20$

$= 58.25$ J

$= 58.3$ J

b) Work Done = Force × Distance

$58.25 = 20R$

$R = \frac{58.25}{20} = 2.9125$

$R = 2.91$ N

c) More realistic if air resistance is considered to vary with speed.

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7. [*In this question use* $g = 10\,m\,s^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,m\,s^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,m\,s^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,m\,s^{-1}$.

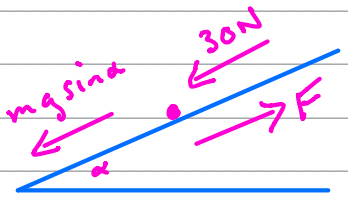
(5)

a) Power $= Fv$

$= 30 \times 4$

$= 120$ Watts

b)



$\sin\alpha = \frac{1}{15}$

Constant speed so $F = mg\sin\alpha + 30$

$F = 60 \times 10 \times \frac{1}{15} + 30$

$F = 70$ N

Power $= Fv = 70 \times 3$

$= 210$ Watts

8. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

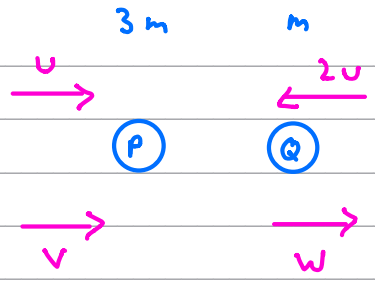
Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

a)



PCLM

$$3mu - 2mu = 3mv + mw$$

$$u = 3v + w \quad (1)$$

NLR

$$e = \frac{w - v}{3u}$$

$$3eu + v = w \quad (2)$$

Sub for w in (1)

$$u = 3v + 3eu + v$$

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Question 8 continued

$$u - 3eu = 4v$$

$$v = \frac{u(1-3e)}{4}$$

b) P changes direction, $\therefore v < 0$

$$\Rightarrow 1 - 3e < 0$$

$$\frac{1}{3} < e \leq 1$$

c) From (2)

$$w = 3eu + v$$

$$w = 3eu + \frac{u(1-3e)}{4}$$

$$w = \frac{12eu + u - 3eu}{4}$$

$$w = \frac{9eu + u}{4}$$

$$w = \frac{u(1+9e)}{4}$$

d) $e = \frac{5}{9} \Rightarrow w = \frac{u}{4}\left(1 + 9 \times \frac{5}{9}\right)$

$$w = \frac{3u}{2}$$

Question 8 continued

After hitting wall speed of Q $= f \times \frac{3}{2}u$

$= \frac{3fu}{2}$

If Q then catches up to collide with P

$\frac{3fu}{2} > |v|$

$\frac{3fu}{2} > \left|\frac{u\left(1 - 3 \times \frac{5}{9}\right)}{4}\right|$

$\frac{3fu}{2} > \left|\frac{u}{4}\left(-\frac{2}{3}\right)\right|$

$\frac{3fu}{2} > \frac{u}{6}$

$f > \frac{2}{3 \times 6}$

$f > \frac{1}{9}$

$\frac{1}{9} < f \leq 1$