Mathematics

Advanced Subsidiary

Paper 1: Pure Mathematics

Paper 1 Pure Mathematics	
You must have: mathematical formulae and statistical tables, calculator	
Time	2 hours

Name	
Class	
Teacher name	

Total marks	/100

1 Find an equation of a line l which passes through $P(-2, 6)$ and $Q(4, -2)$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(Total for Question 1 is 3 marks)

2 $\overrightarrow{AB} = -3k\mathbf{i} + k\mathbf{j}$

The magnitude of $\overrightarrow{AB}$ is $5\sqrt{30}$

Find the possible values of k, leaving your answer in simplified surd form.

(3)

(Total for Question 2 is 3 marks)

3 A freelance journalist charges an initial fixed fee and then an extra fee per word.

She charges £250 for a 300 word article and £410 for a 700 word article.

a Write an equation linking words, w, and fee, F, in the form $F = aw + b$.

(3)

b Interpret the values of a and b.

(2)

She charges a company £650 to write another article.

c Calculate the word length of this article.

(2)

(Total for Question 3 is 7 marks)

4 Given that $y = \frac{16}{81}x^4$, express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{3}{4}}$

(2)

b $\frac{2}{3}y^{-\frac{1}{2}}$

(2)

(Total for Question 4 is 4 marks)

5 Point P lies on the line with equation $2x - y - 5 = 0$.

Point P is a distance of $\sqrt{130}$ from the origin.

Show that there are two possible positions for point P and find the coordinates for each of these points.

Show each step of your working.

(5)

(Total for Question 5 is 5 marks)

6 A company expects to sell 20000 computers in the first year if the price of each computer is £650.

Let x represent the number of £'s by which the price has decreased.

a Write an expression for the price, p, of one computer, in the form $p = a + bx$.

(1)

The company expects to sell an additional 50 computers every time the price decreases by £1.

b Write an expression for the number of computers sold, C, in the form $C = d + ex$.

(1)

Revenue is defined by the formula,

revenue = (number of computers sold) × (cost of one computer)

c Write an equation for revenue, r, in the form $A - B(x - C)^2$, where A, B and C are constants to be found.

(4)

The company wishes to maximise the revenue.

d Using your answer to part **c**, or othwerwise, state the price the company should charge for each computer and the revenue they will attain.

(2)

(Total for Question 6 is 8 marks)

7 The points $P(-5, -13)$ and $Q(7, 3)$ lie on a circle C with centre $(a, -8)$ and radius r.

Find the equation of the circle C.

(8)

(Total for Question 7 is 8 marks)

8 The equation $kx^2 - 3kx + 15 = 0$, where k is a constant, has two real roots.

Prove that $k < 0$ or $k > \frac{20}{3}$.

(3)

(Total for Question 8 is 3 marks)

9 $f(x) = x^2 - 7x + 10$

$g(x) = 6 - 2x$

a Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes.

(4)

b Find the coordinates of any points of intersection.

(4)

c Write down the sets of values of x for which $g(x) > f(x)$.

(1)

(Total for Question 9 is 9 marks)

10 Figure 1 shows a triangle, ABC.

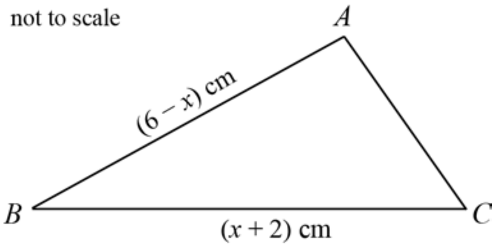

Figure 1

$\angle ABC = 30°$

$AB = (6 - x)$ cm

$BC = (x + 2)$ cm.

The area of the triangle is A cm^2.

a Show that $A = \frac{1}{4}\left(-x^2 + 4x + 12\right)$.

(3)

b Find the maximum value of A and the value of x at which it occurs.

(4)

(Total for Question 10 is 7 marks)

11 Prove that, for any positive numbers a and b, where $a \neq b$, $a^2 + b^2 > 2ab$.

(3)

(Total for Question 11 is 3 marks)

12 In the binomial expansion of $(1 + px)^8$, the coefficient of x^3 is 252 times the coefficient of x.

Find the value of the coefficient of x^2.

(5)

(Total for Question 12 is 5 marks)

13 Solve for $-180^\circ \leqslant x < 180^\circ$, $8\cos^2 x + 10\cos x = 13 - 5\sin^2 x$.

Give your answers to one decimal place.

(5)

(Total for Question 13 is 5 marks)

14 Prove, from first principles, that the derivative of $4x^3$ is $12x^2$.

(4)

(Total for Question 14 is 4 marks)

15 The value, V in £'s, of a car t years after purchase can be modelled by the equation,

$V = 28\,000e^{-0.19t} + 2000$ for $t \geqslant 0$

a State the initial value of the car.

(1)

b Interpret the meaning of the 2000 in the model.

(1)

c Find $\frac{dV}{dt}$ and state how $\frac{dV}{dt}$ shows the value of the car decreases over time.

(2)

d Show that, when the value of the car is £18 000, $t = \frac{100}{19}\ln\left(\frac{7}{4}\right)$.

(4)

(Total for Question 15 is 8 marks)

16 Figure 2 shows a line with equation $x + y = 11$.

It intersects a curve with equation $y = -\frac{1}{2}x^2 + 4x + 3$ at the points P and Q.

The shaded region R_1 is a trapezium bounded by PQ, the x-axis and lines parallel to the y-axis through P and Q.

The shaded region R_2 is the finite region bounded by the line and the curve.

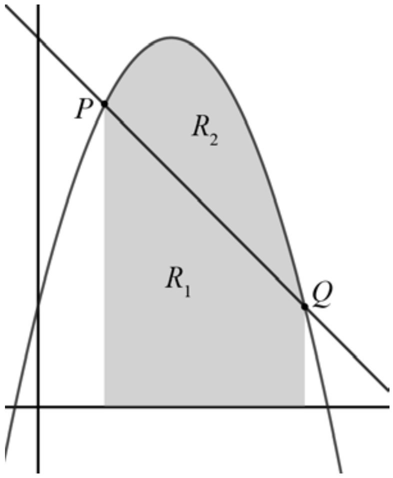

Figure 2

Show that the areas of the shaded regions R_1 and R_2 are in the ratio 2:1.

(8)

(Total for Question 16 is 8 marks)

17 Figure 3 shows the plan view of a garden where part of the garden has been enclosed with 250 m of fencing.

The shape of the enclosed part of the garden is a rectangular section joined to a semicircular section.

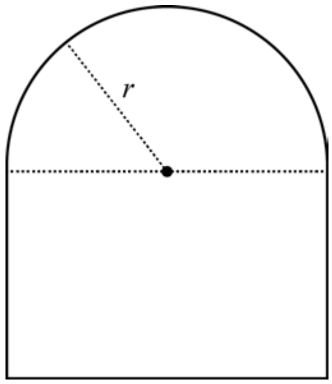

Figure 3

Given that the radius of the semicircular section is r metres, show that,

a the area, A m^2, of the enclosed part of the garden is given by $A = 250r - \left(\frac{4+\pi}{2}\right)r^2$

(5)

b the maximum value of the area of the enclosed part of the garden is $A = \frac{250^2}{2(4+\pi)}$

(5)

(Total for Question 17 is 10 marks)

TOTAL FOR PAPER IS 100 MARKS