$$3x + 4y = 18$$
$$2x - y = 1$$

$$\begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 1 \end{pmatrix}$$

$$\underline{M} = \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix} \qquad \underline{M}^{-1} = \frac{1}{\det \underline{M}}\begin{pmatrix} -1 & -4 \\ -2 & 3 \end{pmatrix}$$

$$= \frac{1}{-11}\begin{pmatrix} -1 & -4 \\ -2 & 3 \end{pmatrix}$$

$$\underline{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 1 \end{pmatrix}$$

$$\underline{M}^{-1}\underline{M}\begin{pmatrix} x \\ y \end{pmatrix} = \underline{M}^{-1}\begin{pmatrix} 18 \\ 1 \end{pmatrix}$$

$$\underline{I}_2\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} 18 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{11}\begin{pmatrix} -1 & -4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 18 \\ 1 \end{pmatrix} = -\frac{1}{11}\begin{pmatrix} -18-4 \\ -36+3 \end{pmatrix}$$

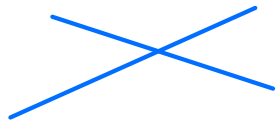
$$= -\frac{1}{11}\begin{pmatrix} -22 \\ -33 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

With 2 linear eqns in 2 unknowns there are 3 possibilities

1) Unique solution

$$3x + 4y = 18$$
$$2x - y = 1$$



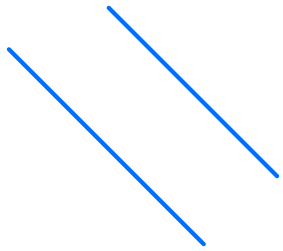
2)

$$x + 2y = 10$$
$$2x + 4y = 30$$

Parallel Lines

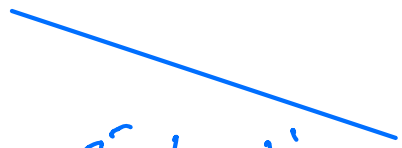
$$\underline{M} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

$$\det \underline{M} = 4 - 4 = 0$$



3)

$$3x + 2y = 10$$
$$6x + 4y = 20$$



single line
(both the same line)

$$\underline{M} = \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$$

$$\det \underline{M} = 12 - 12 = 0$$

1) eqns consistent unique solution

$\det \underline{M} \neq 0$ coefficient matrix non-singular

2) eqns inconsistent no solution

$\det \underline{M} = 0$ coefficient matrix singular

3) eqns consistent infinite amount of solutions

$\det \underline{M} = 0$ coefficient matrix singular

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Exercise 6F

1 a)

$$2x - 6y + 4z = 32$$
$$3x + 2y - 9z = -49$$
$$-2x + 4y + z = -3$$

$$\underline{M} = \begin{pmatrix} 2 & -6 & 4 \\ 3 & 2 & -9 \\ -2 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{M}^{-1} \begin{pmatrix} 32 \\ -49 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix}$$

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Manual Solution

$$2x - 6y + 4z = 32 \quad (1)$$

$$3x + 2y - 9z = -49 \quad (2)$$

$$-2x + 4y + z = -3 \quad (3)$$

(2) × 3 $\quad 9x + 6y - 27z = -147 \quad (4)$

(1) + (4) $\quad 11x - 23z = -115 \quad (5)$

(2) × 2 $\quad 6x + 4y - 18z = -98 \quad (6)$

(6) − (3) $\quad 8x - 19z = -95 \quad (7)$

(7) × 11 $\quad 88x - 209z = -1045 \quad (8)$

(5) × 8 $\quad 88x - 184z = -920 \quad (9)$

(8) − (9) $\quad -25z = -125$

$$\underline{z = 5}$$

Sub in (7) $\quad 8x - 19 \times 5 = -95$

$$8x - 95 = -95$$

$$8x = 0 \qquad \underline{x = 0}$$

Sub in ①

$$2(0) - 6y + 4(5) = 32$$

$$-6y + 20 = 32$$

$$-6y = 12$$

$$\underline{y = -2}$$

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Hwk Q3, Q4 Exercise 6F