Exercise 7E Transformations

$$\underline{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ -z \\ y \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 \\ 0 & \sin 90 & \cos 90 \end{pmatrix}$$

Rotation $90°$ anti-clockwise about x-axis

(looking from +ve x-axis back to origin)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ -a \\ 2a-1 \end{pmatrix} = \begin{pmatrix} a \\ 1-2a \\ -a \end{pmatrix} = \begin{pmatrix} a \\ a-5 \\ -a \end{pmatrix}$$

$$\Rightarrow \quad 1 - 2a = a - 5$$

$$6 = 3a$$

$$\underline{a = 2}$$

Exercise 7F

1) $\underline{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

a) rotation by $90°$ anti-clockwise about $(0,0)$

b) clockwise $90°$ about $(0,0)$ would be inverse

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

c) rotation by $90°$ clockwise about $(0,0)$

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3) $\underline{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \underline{B} = \begin{pmatrix} \cos 270 & -\sin 270 \\ \sin 270 & \cos 270 \end{pmatrix}$

$$\underline{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\underline{C} = \underline{B}\,\underline{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$ reflection in x-axis

b) $C^{-1} = C$ reflect in x-axis again

c) $\underline{D} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\underline{D} = \underline{A}\,\underline{B}$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$

reflection in y-axis

$\underline{D}^{-1} = \underline{D}$ reflect in y-axis again

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5) $\underline{E} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

a) Enlargement about $(0,0)$ s.f. 4

$$\underline{E}^{-1} = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$$

$$\underline{E}\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix} = \begin{pmatrix} 4 & 9 & 3 \\ 6 & 7 & 1 \end{pmatrix}$$

$$E^{-1}E\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix} = E^{-1}\begin{pmatrix} 4 & 9 & 3 \\ 6 & 7 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 4 & 9 & 3 \\ 6 & 7 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 9/4 & 3/4 \\ \frac{3}{2} & \frac{7}{4} & \frac{1}{4} \end{pmatrix}$$

$$\left(1, \frac{3}{2}\right) \quad \left(9/4, 7/4\right) \quad \left(3/4, 1/4\right)$$
