Quadratic sequences

We have seen that if the 2nd differences are all equal to 2, the n^2 is in the formula for the n^{th} term.

If the 2nd differences are all equal to 4 then $2n^2$ will be in the formula

2nd diff	number of n^2
1	$\frac{1}{2}n^2$
2	n^2
4	$2n^2$
6	$3n^2$

Ex

2nd diff		4	4	4	
1st diff	9	13	17	21	
	9	18	31	48	69
$2n^2$	2	8	18	32	50
	7	10	13	16	19
$+3n$	3	6	9	12	15
$+4$	4	4	4	4	4

n^{th} term $= 2n^2 + 3n + 4$

Ex 2

2nd diff		6	6	6	
1st diff	7	13	19	25	
	2,	9,	22,	41,	66
$3n^2$	3	12	27	48	75
	-1	-3	-5	-7	-9
$-2n$	-2	-4	-6	-8	-10
$+1$	+1	+1	+1	+1	+1

n^{th} term $= 3n^2 - 2n + 1$

Exercise Find n^{th} term

1) 4 15 30 49 72

2) 9 18 29 42 57

3) 11 21 37 59 87

Solutions

1)

2nd diff		4	4	4	
1st diff	11	15	19	23	
	4	15	30	49	72
$2n^2$	2	8	18	32	50
	2	7	12	17	22

$+5n$	5	10	15	20	25
-3	-3	-3	-3	-3	-3

n^{th} term $= 2n^2 + 5n - 3$

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2)

2nd diff		2	2	2	
1st diff	9	11	13	15	
	9	18	29	42	57
n^2	1	4	9	16	25
	8	14	20	26	32
$+6n$	6	12	18	24	30
$+2$	2	2	2	2	2

n^{th} term $= n^2 + 6n + 2$

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3)

2nd diff		6	6	6	
1st diff	10	16	22	28	
	11	21	37	59	87
$3n^2$	3	12	27	48	75
	8	9	10	11	12
$+n$	1	2	3	4	5
$+7$	7	7	7	7	7

n^{th} term $= 3n^2 + n + 7$

Fibonacci Sequences

The standard Fibonacci sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

A term is obtained by adding together the previous terms.

Two terms are required to start the sequence.

Other examples

4, 9, 13, 22, 35, 57, ...

5, 12, 17 29, 46, 75, ...

6, 7, 13, 20, 33, 53, ...

Geometric Sequences

A geometric sequence is obtained by multiplying every term by the same amount to find successive terms

Examples

1) 1, 2, 4, 8, 16, 32 ... ×2

2) 32, 16, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, ... ×$\frac{1}{2}$

3) 4, 12, 36, 108, 324, ... ×3

4) 1, 5, 25, 125, 625, ... ×5

There is a relationship between
3 successive terms