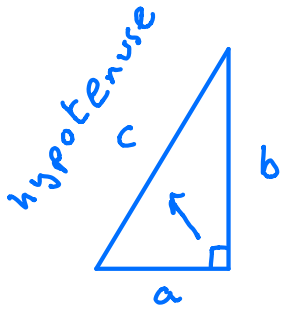
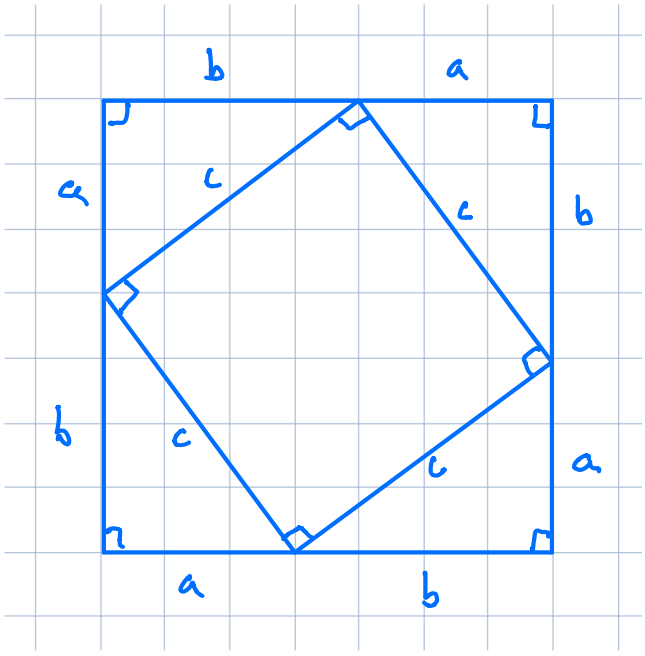
Pythagoras Theorem (for right-angled triangles)



$$a^2 + b^2 = c^2$$

where c is the hypotenuse
(the longest side opposite the right-angle)



Proof of Pythagoras Theorem

Layout 4 copies of a right-angled triangle as above

whole shape is a square with side $a+b$

$$\therefore \text{ area } = (a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + b^2 + 2ab$$

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whole shape is also the square in the middle + 4 triangles

$$\text{Area} = c^2 + 4 \times \tfrac{1}{2}ab$$

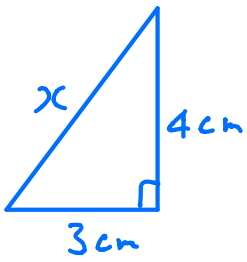
$$= c^2 + 2ab$$

Shape can only have one area

$$\therefore \quad a^2 + b^2 + \cancel{2ab} = c^2 + \cancel{2ab}$$

$$\Rightarrow \quad a^2 + b^2 = c^2$$

Ex1



By Pythagoras

$$3^2 + 4^2 = x^2$$

$$9 + 16 = x^2$$

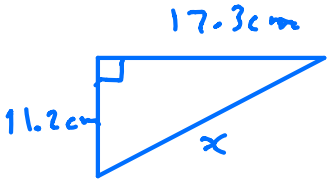
$$25 = x^2$$

$$\sqrt{25} = x$$

$$x = 5\text{cm}$$

3, 4, 5 right-angled triangle

Ex2



By Pythagoras

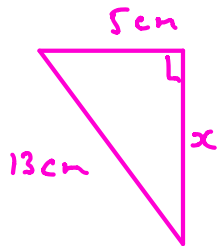
$$11.2^2 + 17.3^2 = x^2$$

$$424.73 = x^2$$

$$\sqrt{424.73} = x$$

$$x = 20.6\text{cm}$$

Ex 3



By Pythagoras

$x^2 + 5^2 = 13^2$

$x^2 = 13^2 - 5^2$

$x^2 = 169 - 25$

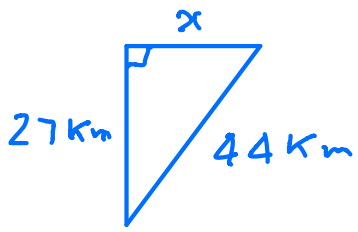
$x^2 = 144$

$x = \sqrt{144}$

$x = 12\text{cm}$

5, 12, 13 right-angled triangle

Ex 4



By Pythagoras

$x^2 + 27^2 = 44^2$

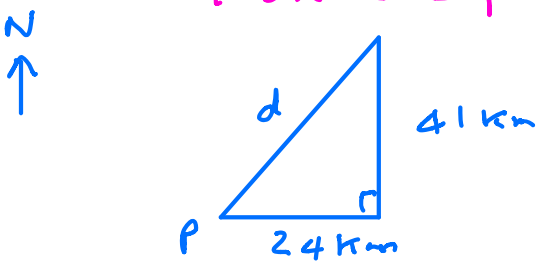
$x^2 = 44^2 - 27^2$

$x^2 = 1207$

$x = \sqrt{1207}$

$x = 34.7\text{ km}$

Ex 5 A ship leaves port and sails 24 km East. It then sails 41 km North. How far is the ship from the port.



By Pythagoras

$24^2 + 41^2 = d^2$

$2257 = d^2$

$\sqrt{2257} = d$

$d = 47.5\text{ km}$

Ship is 47.5 km from the port