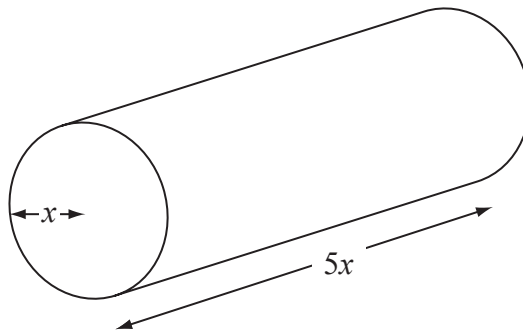**3.**

Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm. The cross-sectional area of the rod is increasing at the constant rate of 0.032 cm² s⁻¹.

(a) Find $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

a)   Cross - Section $A = \pi x^2$ $\Rightarrow \dfrac{dA}{dx} = 2\pi x$

$$\frac{dx}{dt} = \frac{dx}{dA} \times \frac{dA}{dt} = \frac{1}{\frac{dA}{dx}} \times \frac{dA}{dt}$$

$$\frac{dx}{dt} = \frac{1}{2\pi x} \times 0.032 \quad cm\,s^{-1}$$

When $x = 2cm$, $\dfrac{dx}{dt} = \dfrac{1}{2\pi \times 2} \times 0.032$

$$\frac{dx}{dt} = 0.00255 \quad cm\,s^{-1} \qquad 3\,s.f.$$

Question 3 continued

b) $\quad V = \pi x^2 \times 5x = 5\pi x^3 \qquad \Rightarrow \dfrac{dV}{dx} = 15\pi x^2$

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$$

When $x = 2$

$$\frac{dV}{dt} = 15\pi \times 2^2 \times 0.0025465$$

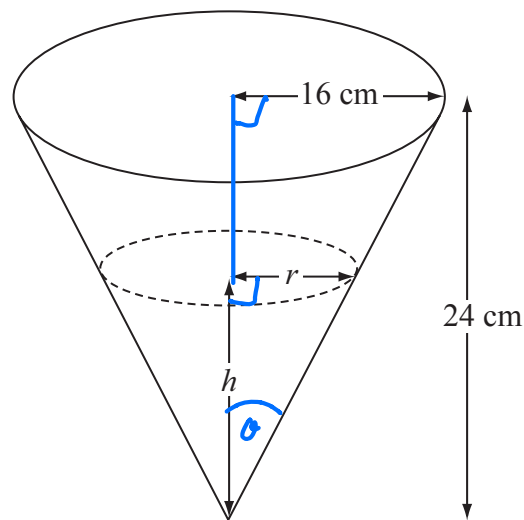$$\frac{dV}{dt} = 0.480 \ cm^3 \ s^{-1} \qquad 3 \ s.f.$$

5.

Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

a)

$$\tan\theta = \frac{r}{h} = \frac{16}{24}$$

$$\Rightarrow \quad 24r = 16h$$

$$r = \frac{2h}{3}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{4\pi h^3}{27}$$

Question 5 continued

b) $\dfrac{dV}{dt} = 8 \text{ cm}^3 s^{-1}$ Find $\dfrac{dh}{dt}$ when $h = 12$

$$V = \dfrac{4\pi h^3}{27} \quad \Rightarrow \quad \dfrac{dV}{dh} = \dfrac{4\pi h^2}{9}$$

$$\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt}$$

$$= \dfrac{1}{\dfrac{dV}{dh}} \times \dfrac{dV}{dt}$$

$$\dfrac{dh}{dt} = \dfrac{1}{\dfrac{4\pi h^2}{9}} \times 8$$

When $h = 12$ $\dfrac{dh}{dt} = \dfrac{72}{4\pi \times 12^2}$

$$\dfrac{dh}{dt} = \dfrac{1}{8\pi} \text{ cm } s^{-1}$$

6. The area A of a circle is increasing at a constant rate of $1.5\,\text{cm}^2\,\text{s}^{-1}$. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is $2\,\text{cm}^2$.

(5)

$$A = \pi r^2 \qquad \Rightarrow \qquad \frac{dA}{dr} = 2\pi r$$

Find $\dfrac{dr}{dt}$ when $A = 2\,\text{cm}^2$

If $A = 2\,\text{cm}^2 \qquad \pi r^2 = 2$

$$r = \sqrt{\frac{2}{\pi}}$$

$$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt} \qquad\qquad \text{Given } \frac{dA}{dt} = 1.5\,\text{cm}^2\,\text{s}^{-1}$$

$$\frac{dr}{dt} = \frac{1}{\frac{dA}{dr}} \times \frac{dA}{dt}$$

When $A = 2\,\text{cm}^2 \qquad\qquad \dfrac{dr}{dt} = \dfrac{1}{2\pi r} \times 1.5$

$$\frac{dr}{dt} = \frac{1}{2\pi \sqrt{\frac{2}{\pi}}} \times 1.5$$

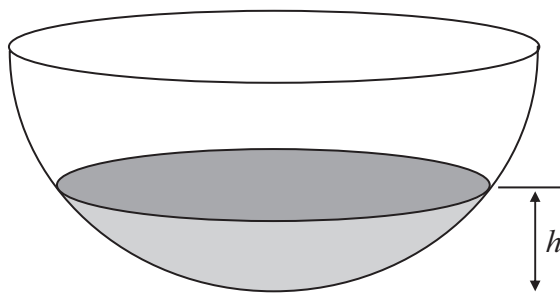$$\frac{dr}{dt} = 0.299 \ \text{cm s}^{-1}$$

3.

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.
When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2 (3 - 4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\dfrac{dV}{dh}$ when $h = 0.1$

(4)

Water flows into the bowl at a rate of $\dfrac{\pi}{800}$ m^{3}s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$

(2)

a)

$$V = \frac{3\pi h^2}{12} - \frac{4\pi h^3}{12}$$

$$\frac{dV}{dh} = \frac{6\pi h}{12} - \frac{12\pi h^2}{12}$$

$$\frac{dV}{dh} = \frac{\pi h}{2} - \pi h^2$$

when $h = 0.1$

$$\frac{dV}{dh} = \frac{\pi}{2} \times 0.1 - \pi \times 0.1^2$$

$$\frac{dV}{dh} = \frac{\pi}{25}$$

Leave blank

Question 3 continued

b)   Given $\dfrac{dV}{dt} = \dfrac{\pi}{800}$ $m^3 s^{-1}$

Find $\dfrac{dh}{dt}$ when $h = 0.1$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$= \frac{1}{\dfrac{dV}{dh}} \times \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\dfrac{\pi}{25}} \times \frac{\pi}{800}$$
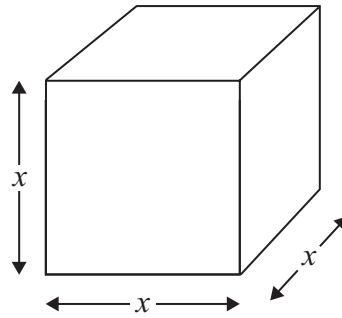
$$\frac{dh}{dt} = \frac{1}{32} \; m s^{-1}$$

Leave
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2.

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.
At time t seconds, the length of each edge of the cube is x cm,
and the volume of the cube is V cm^3.

(a) Show that $\dfrac{dV}{dx} = 3x^2$

(1)

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^{3}s^{-1},

(b) find $\dfrac{dx}{dt}$, when $x = 8$

(2)

(c) find the rate of increase of the total surface area of the cube, in cm^{2}s^{-1},
     when $x = 8$

(3)

a) $V = x \times x \times x = x^3$

$\dfrac{dv}{dx} = 3x^2$

b)   Given $\dfrac{dV}{dt} = 0.048 \ cm^3 s^{-1}$

Find $\dfrac{dx}{dt}$ when $x = 8$

$\dfrac{dx}{dt} = \dfrac{dx}{dV} \times \dfrac{dV}{dt} = \dfrac{1}{\frac{dV}{dx}} \times \dfrac{dV}{dt}$

6

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Question 2 continued

$$\frac{dx}{dt} = \frac{1}{3x^2} \times 0.048 \quad cm\,s^{-1}$$

When $x = 8$,

$$\frac{dx}{dt} = \frac{1}{3 \times 8^2} \times 0.048 \quad cm\,s^{-1}$$

$$\frac{dx}{dt} = 0.00025 \quad cm\,s^{-1}$$

c) Surface Area $S = 6x^2$

$$\frac{dS}{dx} = 12x$$

$$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$$

$$= 12x \times 0.00025$$

$$= 12 \times 8 \times 0.00025$$

$$\frac{dS}{dt} = 0.024 \quad cm^2\,s^{-1}$$
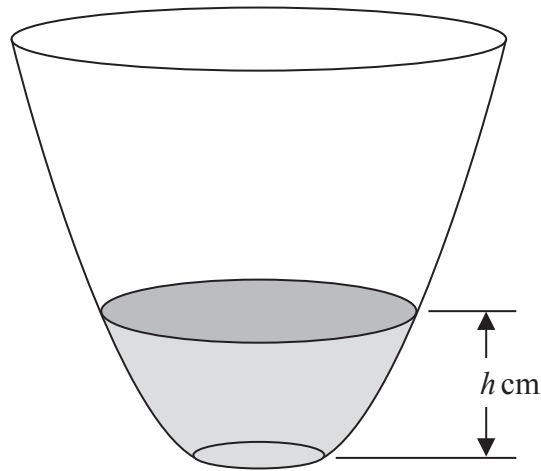
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4.

Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm^3 is given by

$$V = 4\pi h(h + 4), \quad 0 \leqslant h \leqslant 25$$

Water flows into the vase at a constant rate of 80π cm^{3}s^{-1}

Find the rate of change of the depth of the water, in cms^{-1}, when $h = 6$

(5)

$$V = 4\pi h^2 + 16\pi h$$

$$\frac{dV}{dh} = 8\pi h + 16\pi \quad = 8\pi\left(h + 2\right)$$

Given $\frac{dV}{dt} = 80\pi$ cm^3 s^{-1}

Find $\frac{dh}{dt}$ when $h = 6$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$= \frac{1}{\frac{dV}{dh}} \times \frac{dV}{dt}$$

Question 4 continued

$$\frac{dh}{dt} = \frac{1}{8\pi(h+2)} \times 80\pi$$

$h = 6,$

$$\frac{dh}{dt} = \frac{10}{6+2}$$

$$\frac{dh}{dt} = 1.25 \text{ cm s}^{-1}$$