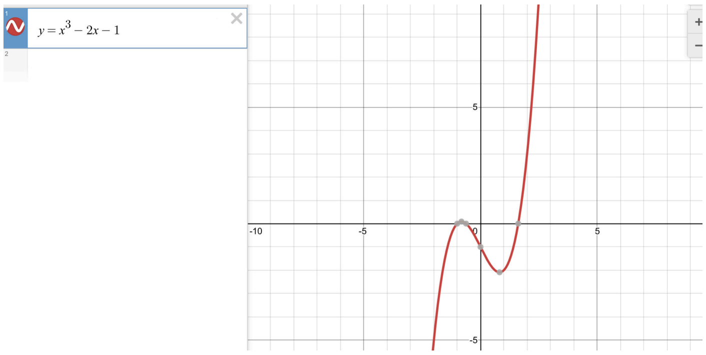
Approximate Solutions to Equations



Show the eqn

$x^3 - 2x - 1 = 0$

has a solution between

$x = 1$ and $x = 2$

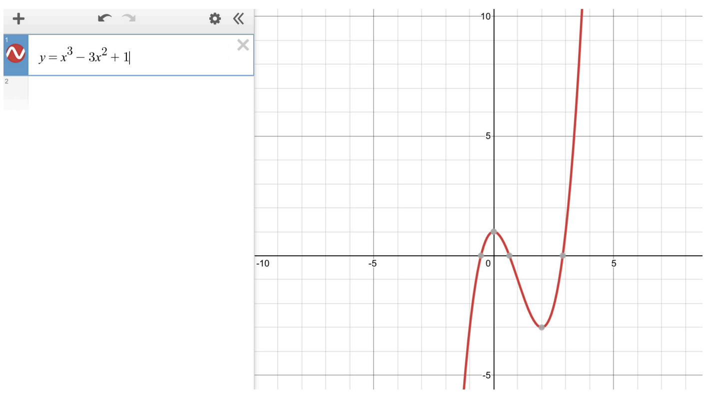
$f(x) = x^3 - 2x - 1$

$f(1) = 1^3 - 2(1) - 1 = -2$

$f(2) = 2^3 - 2(2) - 1 = +3$

We have a sign change in the value of the function between $x = 1$ and $x = 2$. Since the function is continuous $f(x) = 0$ has a solution between $x = 1$ and $x = 2$

Ex 2



Let $f(x) = x^3 - 3x^2 + 1$

Show $f(x) = 0$

has a solution between

$x = 2$ and $x = 3$

$$f(2) = 2^3 - 3(2)^2 + 1$$
$$= 8 - 12 + 1 = -3$$

$$f(3) = 3^3 - 3(3)^2 + 1$$
$$= 27 - 27 + 1 = +1$$

Continuous function has sign change between $x = 2$ and $x = 3$ so $f(x) = 0$ has a solution between $x = 2$ and $x = 3$

Typical Exam Question

a) Show the eqn $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

when $x = 0$ $\quad 0^3 + 4(0) = 0 < 1$

when $x = 1$ $\quad 1^3 + 4(1) = 5 > 1$

$x^3 + 4x$ is a continuous function so $x^3 + 4x = 1$ for some value of x between 0 and 1

b) Show that the eqn $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$$x^3 + 4x = 1$$

$$4x = 1 - x^3$$

$$x = \frac{1}{4} - \frac{x^3}{4}$$

c) Starting with $x_0 = 0$ use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice

to find an estimate for the solution of
$$x^3 + 4x = 1$$

$x_0 = 0$

$$x_1 = \frac{1}{4} - \frac{0^3}{4} = \frac{1}{4} - 0 = \frac{1}{4}$$

$$x_2 = \frac{1}{4} - \frac{\left(\frac{1}{4}\right)^3}{4} = 0.246$$