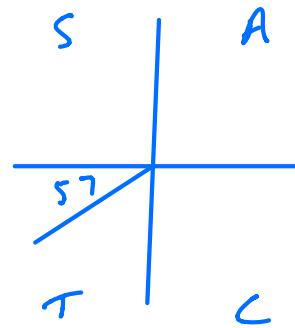


Mixed Exercise 10

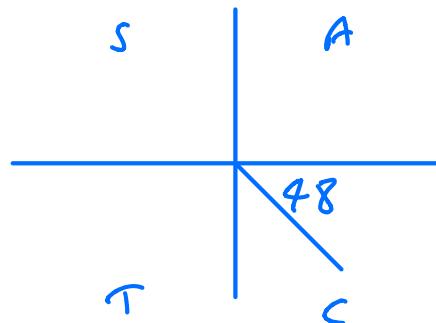
1a $\cos 237^\circ$

$$= -\cos 57^\circ$$



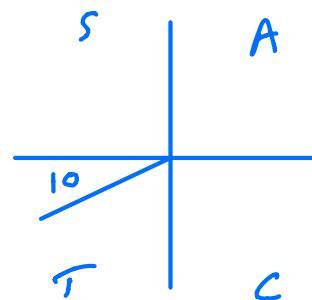
1b $\sin 312^\circ$

$$= -\sin 48^\circ$$



1c $\tan 190^\circ$

$$= \tan 10^\circ$$



3) $\cos A = -\sqrt{\frac{7}{11}}$ show $\tan A = -\frac{2\sqrt{7}}{7}$

A obtuse

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin^2 A = 1 - \frac{7}{11}$$

$$\sin^2 A = \frac{4}{11}$$

$$\sin A = \pm \frac{2}{\sqrt{11}} \quad (\text{+ve since } A \text{ is obtuse})$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{2}{\sqrt{11}}}{-\frac{\sqrt{2}}{\sqrt{11}}}$$

$$= \frac{2}{\sqrt{11}} \times -\frac{\sqrt{11}}{\sqrt{2}}$$

$$= -\frac{2}{\sqrt{2}}$$

$$= -\frac{2\sqrt{2}}{2}$$

5) a) $\cos^4 \theta - \sin^4 \theta$

$$(\cos^2 \theta)^2 - (\sin^2 \theta)^2$$

$$= (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)$$

$$= 1(\cos^2 \theta - \sin^2 \theta)$$

$$= \cos^2 \theta - \sin^2 \theta$$

5)

$$\sin^2 3\theta - \sin^2 3\theta \cos^2 3\theta$$

$$\sin^2 3\theta (1 - \cos^2 3\theta)$$

$$\sin^2 3\theta \sin^2 3\theta$$

$$= \sin^4 3\theta$$

$$\begin{aligned}
 & \text{c) } \cos^4\theta + 2\sin^2\theta \cos^2\theta + \sin^4\theta \\
 & \quad (\cos^2\theta + \sin^2\theta)^2 \\
 & \quad = 1^2 \\
 & \quad = 1
 \end{aligned}$$

7 a) Prove $(1+\sin\theta)^2 + \cos^2\theta = 2(1+\sin\theta)$

$$\begin{aligned}
 (1+\sin\theta)^2 + \cos^2\theta & \equiv 1 + 2\sin\theta + \sin^2\theta + \cos^2\theta \\
 & \equiv 1 + 2\sin\theta + 1 \\
 & \equiv 2 + 2\sin\theta \\
 & \equiv 2(1+\sin\theta)
 \end{aligned}$$

b) Prove $\cos^4\theta + \sin^4\theta \equiv \sin^4\theta + \cos^2\theta$

$$\begin{aligned}
 \cos^4\theta + \sin^4\theta & \equiv \cos^2\theta \cos^2\theta + \sin^2\theta \sin^2\theta \\
 & \equiv \cos^2\theta (1 - \sin^2\theta) + \sin^2\theta \\
 & \equiv \cos^2\theta - \sin^2\theta \cos^2\theta + \sin^2\theta \\
 & \equiv 1 - \sin^2\theta \cos^2\theta \quad *
 \end{aligned}$$

$$\begin{aligned}
 \text{Also } & \sin^4\theta + \cos^2\theta \\
 = & \sin^2\theta \sin^2\theta + \cos^2\theta \\
 = & \sin^2\theta(1 - \cos^2\theta) + \cos^2\theta \\
 = & \sin^2\theta - \sin^2\theta \cos^2\theta + \cos^2\theta \\
 = & 1 - \sin^2\theta \cos^2\theta \quad * *
 \end{aligned}$$

* is the same **

$$\therefore \cos^4\theta + \sin^2\theta = \sin^4\theta + \cos^2\theta$$

$$\begin{aligned}
 1) \text{ a) } & 4xy - y^2 + 4x - y \\
 = & y(4x - y) + (4x - y) \\
 = & (y+1)(4x-y)
 \end{aligned}$$

$$0^\circ \leq \theta \leq 360^\circ$$

$$5) \quad 4\sin\theta \cos\theta - \cos^2\theta + 4\sin\theta - \cos\theta = 0$$

$$\text{Let } \sin\theta = x, \cos\theta = y$$

$$\Rightarrow (\cos\theta + 1)(4\sin\theta - \cos\theta) = 0$$

$$\text{Either } \cos\theta + 1 = 0$$

$$\cos\theta = -1$$

$$\underline{\theta = 180^\circ}$$

$$\text{or } 4 \sin \theta - \cos \theta = 0$$

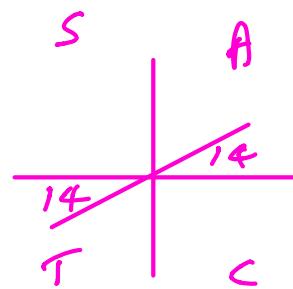
$$4 \sin \theta = \cos \theta$$

$$\frac{4 \sin \theta}{\cos \theta} = 1$$

$$\tan \theta = \frac{1}{4}$$

$$\tan^{-1} \frac{1}{4} = 14.0^\circ$$

$$\begin{array}{r} \theta = 14.0^\circ \\ \hline \theta = 194.0^\circ \end{array}$$

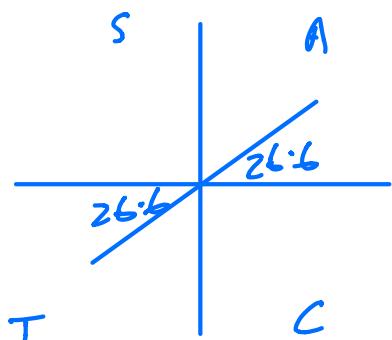


ii) a) $2 \sin 2\theta = \cos 2\theta$

$$\Rightarrow \frac{2 \sin 2\theta}{\cos 2\theta} = 1$$

$$\tan 2\theta = \frac{1}{2}$$

b) $\tan 2\theta = \frac{1}{2}$ $\tan^{-1} \frac{1}{2} = 26.6^\circ$



$$2\theta = 26.6, 206.6, 386.6, 566.6$$

$$\theta = 13.3^\circ, 103.3^\circ, 193.3^\circ, 283.3^\circ$$

(13)

$$\frac{\cos 2x + 0.5}{1 - \cos 2x} = 2$$

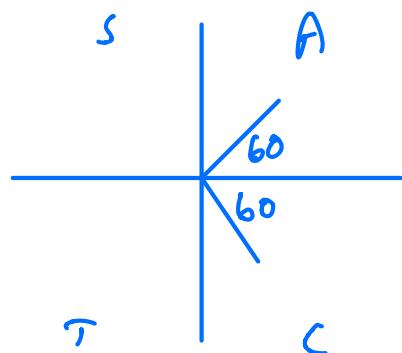
$$0 < x < 270^\circ$$

$$\cos 2x + 0.5 = 2 - 2\cos 2x$$

$$3\cos 2x = 1.5$$

$$\cos 2x = \frac{1.5}{3} = \frac{1}{2}$$

$$\cos^{-1} \frac{1}{2} = 60^\circ$$



$$2x = 60^\circ, 300^\circ, 420^\circ$$

$$x = 30^\circ, 150^\circ, 210^\circ$$
