Leave blank

2. The sequence of positive numbers $u_1, u_2, u_3, \ldots$ is given by:

$$u_{n+1} = (u_n - 3)^2, \qquad u_1 = 1.$$

(a) Find u_2, u_3 and u_4.

(3)

(b) Write down the value of u_{20}.

(1)

a) $U_2 = (1-3)^2 = 4$

$U_3 = (4-3)^2 = 1$

$U_4 = (1-3)^2 = 4$

b) $U_{20} = 4$

Q2

(Total 4 marks)

Leave blank

4. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$
$$a_{n+1} = 3a_n - 5, \quad n \geqslant 1.$$

(a) Find the value of a_2 and the value of a_3.

(2)

(b) Calculate the value of $\sum_{r=1}^{5} a_r$.

(3)

a) $a_2 = 3(3) - 5 = 4$

$a_3 = 3(4) - 5 = 7$

b) $a_4 = 3(7) - 5 = 16$

$a_5 = 3(16) - 5 = 43$

$\sum_{r=1}^{5} a_r = 3 + 4 + 7 + 16 + 43 = 73$

$\sum_{r=1}^{5} a_r = 73$

Jun 07

Leave blank

8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5, \qquad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k. **(1)**

(b) Show that $a_3 = 9k + 20$. **(2)**

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10. **(4)**

a) $a_2 = 3k + 5$

b) $a_3 = 3(3k+5) + 5$

$= 9k + 15 + 5 = 9k + 20$

c) $a_4 = 3(9k+20) + 5$

$= 27k + 60 + 5 = 27k + 65$

i) $\sum_{r=1}^{4} a_r = k + 3k+5 + 9k+20 + 27k+65$

$= 40k + 90$

ii) $\sum_{r=1}^{4} a_r = 40k+90 = 10(4k+9)$

10 is a factor since $4k+9$ is an integer

$\therefore \sum_{r=1}^{4} a_r$ is divisible by 10

H 2 6 1 0 7 A 0 1 4 2 4

Leave blank

7. A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(a) Find x_2 in terms of p. **(1)**

(b) Show that $x_3 = 1 + 3p + 2p^2$. **(2)**

Given that $x_3 = 1$,

(c) find the value of p, **(3)**

(d) write down the value of x_{2008}. **(2)**

a) $x_2 = 1(p+1) = p+1$

b) $x_3 = (p+1)(p+p+1)$

$= (p+1)(2p+1)$

$= 2p^2 + 2p + p + 1$

$= 2p^2 + 3p + 1$

c) $x_3 = 1 \Rightarrow 2p^2 + 3p + 1 = 1$

$2p^2 + 3p = 0$

$p(2p+3) = 0$

$\cancel{p = 0}$ or $p = -\frac{3}{2}$

d) odd terms $= 1$, even $= -\frac{1}{2}$ $\quad x_{2008} = -\frac{1}{2}$

N 2 5 5 6 1 A 0 1 0 2 4

Leave blank

5. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geqslant 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a. **(1)**

(b) Show that $x_3 = a^2 - 3a - 3$. **(2)**

Given that $x_3 = 7$,

(c) find the possible values of a. **(3)**

a) $x_2 = a - 3$

b) $x_3 = a(a-3) - 3 = a^2 - 3a - 3$

c) $a^2 - 3a - 3 = 7$

$a^2 - 3a - 10 = 0$

$(a+2)(a-5) = 0$

$a = -2$ or $a = 5$

H29992A0828

Leave blank

7. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geqslant 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k. **(1)**

(b) Show that $a_3 = 4k - 21$. **(2)**

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k. **(4)**

a) $a_2 = 2k - 7$

b) $a_3 = 2(2k-7) - 7$

$= 4k - 14 - 7$

$= 4k - 21$

c) $a_4 = 2(4k-21) - 7$

$= 8k - 42 - 7$

$= 8k - 49$

$\sum_{r=1}^{4} a_r = k + 2k - 7 + 4k - 21 + 8k - 49 = 43$

$15k - 77 = 43$

$15k = 120$

$k = \frac{120}{15} = 8$

$k = 8$

H 3 4 2 6 2 A 0 1 2 2 8

Leave blank

5. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \qquad n \geqslant 1,$$
$$a_1 = 2$$

(a) Find a_2 and a_3, leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$

(2)

a) $a_2 = \sqrt{2^2 + 3} = \sqrt{7}$

$a_3 = \sqrt{\sqrt{7}^2 + 3} = \sqrt{10}$

b) $a_4 = \sqrt{\sqrt{10}^2 + 3} = \sqrt{13}$

$a_5 = \sqrt{\sqrt{13}^2 + 3} = \sqrt{16} = 4$

$a_5 = 4$

H 3 5 3 8 3 A 0 8 2 8