Exercise 6c

$\frac{1}{2}(e^{A}-e^{-A})-\frac{1}{2}(e^{B}-e^{-B})$

$$\sinh A - \sinh B \equiv 2\sinh\left(\frac{A-B}{2}\right)\cosh\left(\frac{A+B}{2}\right)$$

$$2\sinh\left(\frac{A-B}{2}\right)\cosh\left(\frac{A+B}{2}\right)$$

$$= 2 \times \frac{1}{2}\left[e^{\frac{A-B}{2}} - e^{\frac{B-A}{2}}\right]\frac{1}{2}\left[e^{\frac{A+B}{2}} + e^{-\frac{A-B}{2}}\right]$$

$$= \frac{1}{2}\left[e^{\frac{A}{2}} \times e^{-\frac{B}{2}} - e^{\frac{B}{2}}e^{-\frac{A}{2}}\right]\left[e^{\frac{A}{2}} \times e^{\frac{B}{2}} + e^{-\frac{A}{2}} \times e^{-\frac{B}{2}}\right]$$

$$= \frac{1}{2}\left[e^{A} - e^{B} + e^{-B} - e^{-A}\right]$$

$$= \frac{1}{2}\left[(e^{A}-e^{-A}) - (e^{B}-e^{-B})\right]$$

$$= \sinh A - \sinh B$$

2d)

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\cosh 2A = \frac{1 + \tanh^2 A}{1 - \tanh^2 A}$$

3d)

$\cosh 2x$ $\qquad$ $\cosh x = 2$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$$\cosh 2x = 2\cosh^2 x - 1$$

$$\cosh 2x = 2 \times 2^2 - 1$$

$$\cosh 2x = 7$$

5b) $7\sinh x - 5\cosh x = 1$

$$\frac{7}{2}\left(e^{x} - e^{-x}\right) - \frac{5}{2}\left(e^{x} + e^{-x}\right) = 1$$

$$7\left(e^{x} - e^{-x}\right) - 5\left(e^{x} + e^{-x}\right) = 2$$

$$7e^{x} - 7e^{-x} - 5e^{x} - 5e^{-x} = 2$$

$$2e^{x} - 12e^{-x} = 2$$

$\times e^{x}$ $$2e^{2x} - 12 = 2e^{x}$$

$$2e^{2x} - 2e^{x} - 12 = 0$$

$$e^{2x} - e^{x} - 6 = 0$$

$$(e^{x} - 3)(e^{x} + 2) = 0$$

Either $e^{x} - 3 = 0$ or $e^{x} + 2 = 0$

$e^{x} = 3$ $\quad$ $e^{x} = -2$ ✗

$$\underline{x = \ln 3}$$

3f

$$3\sinh^2 x - 13\cosh x + 7 = 0$$

$$3(\cosh^2 x - 1) - 13\cosh x + 7 = 0$$

$$3\cosh^2 x - 13\cosh x + 4 = 0$$

$$(3\cosh x - 1)(\cosh x - 4) = 0$$

$$3\cosh x - 1 = 0 \quad \text{or} \quad \cosh x - 4 = 0$$

$$\cancel{\cosh x = \tfrac{1}{3}} \qquad \cosh x = 4$$

$$\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$x = \ln\left(4 + \sqrt{16 - 1}\right) = \ln\left(4 + \sqrt{15}\right)$$

5h

$$4\cosh x + 13e^{-x} = 11$$

$$2\left(e^x + e^{-x}\right) + 13e^{-x} = 11$$

$$2e^x + 15e^{-x} - 11 = 0$$

$$2e^{2x} - 11e^x + 15 = 0$$

$$(2e^x - 5)(e^x - 3) = 0$$

$$\Rightarrow \quad e^x = \tfrac{5}{2} \quad \text{or} \quad e^x = 3$$

$x = \ln \frac{5}{2}$ $\qquad$ $x = \ln 3$

Do all other parts of Q5