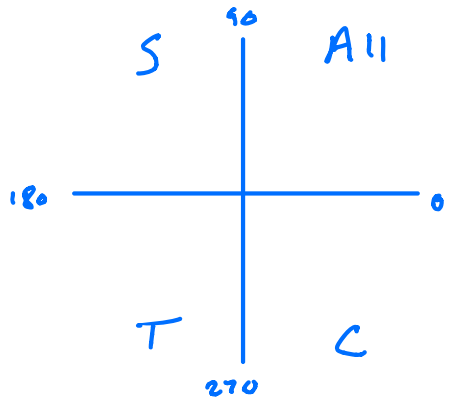
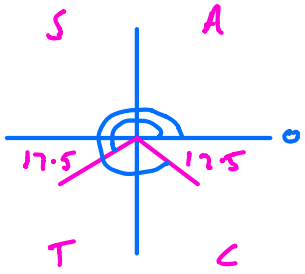
# CAST Diagram and Trigonometric Equations




This diagram shows which functions are positive in each quadrant

Examples Solve $\sin x = -0.3$ for $0 \le x < 360$

1)

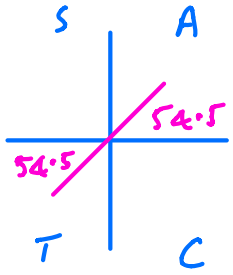



$\sin^{-1} 0.3 = 17.5^\circ$

$x = \sin^{-1}(-0.3)$

$x = 197.5^\circ, 342.5^\circ$

2) Solve $\tan x = 1.4$

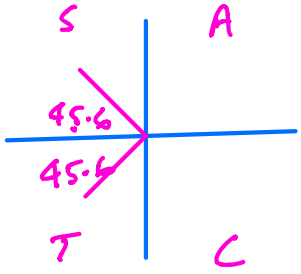



$\tan^{-1}(1.4) = 54.5^\circ$

$x = 54.5^\circ, 234.5^\circ$

3) Solve $\cos x = -0.7$

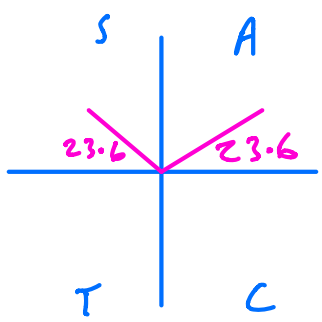
$\cos^{-1}(0.7) = 45.6^\circ$




$x = 134.4^\circ \quad 225.6^\circ$

4) Solve $\sin(2\theta + 15°) = 0.4$ for $0 \le \theta \le 360$

$\sin^{-1} 0.4 = 23.6°$




$2\theta + 15° = 23.6°, 156.4°, 383.6°, 516.4°$

$2\theta = 8.6°, 141.4°, 368.6°, 501.4$

$\theta = 4.3°, 70.7°, 184.3°, 250.7°$

Exercise for $0 \le \theta < 360$ solve

1) $\sin\theta = 0.25$ $\theta = 14.5°, 165.5°$

2) $\cos\theta = -0.63$ $\theta = 129.1°, 230.9°$

3) $\tan\theta = -2.4$ $\theta = 112.6°, 292.6°$

## Trigonometric Equations

**4.** (a) Show that the equation

$$3\sin^2\theta - 2\cos^2\theta = 1$$

can be written as

$$5\sin^2\theta = 3.$$

**(2)**

(b) Hence solve, for $0° \leqslant \theta < 360°$, the equation

$$3\sin^2\theta - 2\cos^2\theta = 1,$$

giving your answers to 1 decimal place.

**(7)**

a)

$$3\sin^2\theta - 2\cos^2\theta = 1$$

$$3\sin^2\theta - 2(1-\sin^2\theta) = 1$$

$$3\sin^2\theta - 2 + 2\sin^2\theta = 1$$

$$\underline{5\sin^2\theta = 3}$$

b)

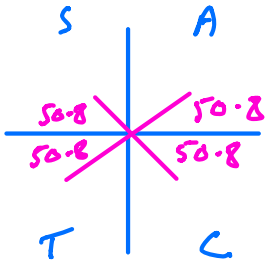
$$3\sin^2\theta - 2\cos^2\theta = 1$$

$$5\sin^2\theta = 3$$

$$\sin^2\theta = \frac{3}{5}$$

$$\sin\theta = \pm\sqrt{\frac{3}{5}}$$

$$\sin^{-1}\sqrt{\frac{3}{5}} = 50.8^\circ$$



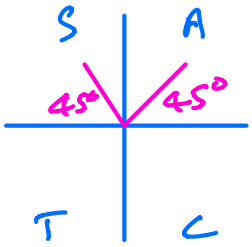

$$\underline{\theta = 50.8^\circ,\ 129.2^\circ,\ 230.8^\circ,\ 309.2^\circ}$$

**9.** Solve, for $0 \leqslant x < 360°$,

(a) $\sin(x - 20°) = \dfrac{1}{\sqrt{2}}$ **(4)**

(b) $\cos 3x = -\dfrac{1}{2}$ **(6)**

a) $\sin(x-20) = \dfrac{1}{\sqrt{2}}$ $\qquad \sin^{-1}\dfrac{1}{\sqrt{2}} = 45^\circ$

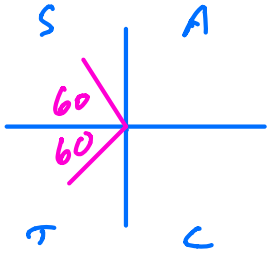



$$x - 20 = 45^\circ,\ 135^\circ$$

$$x = 45 + 20,\ 135 + 20$$

$$x = 65^\circ,\ 155^\circ$$

b) $\cos 3x = -\frac{1}{2}$ $\qquad$ $\cos^{-1}\frac{1}{2} = 60°$




$3x = 120, 240, 480, 600, 840, 960$

$x = 40°, 80°, 160°, 200°\ 280°\ 320°$