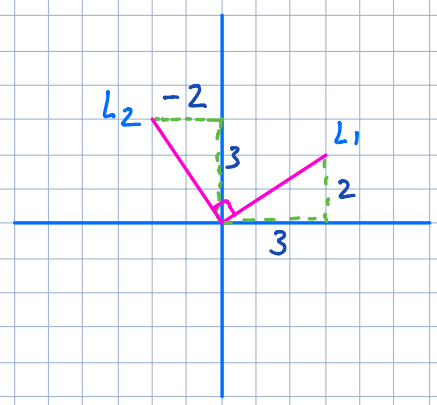
Parallel and Perpendicular Lines



L_1 has gradient $\frac{2}{3}$

When it is rotated 90° to give L_2 we see that L_2 has gradient $-\frac{3}{2}$

In general, if Lines L_1 and L_2 are perpendicular then their gradients m_1 and m_2 are related by,

$$m_1 \times m_2 = -1$$

$$\text{or} \quad m_2 = -\frac{1}{m_1}$$

To find a gradient perpendicular to a given gradient we invert it and change the sign

Examples

	m_1	m_2
1	$\frac{2}{3}$	$-\frac{3}{2}$
2	$-\frac{3}{4}$	$+\frac{4}{3}$
3	5	$-\frac{1}{5}$

4	-1	$+1$
5	$\frac{1}{2}$	-2

Exercise Find perpendicular gradients

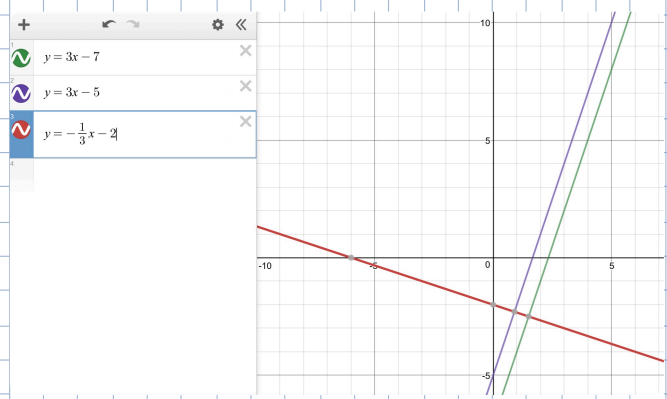
	m_1	m_2
1	4	$-\frac{1}{4}$
2	$-\frac{1}{3}$	$+3$
3	$\frac{2}{5}$	$-\frac{5}{2}$
4	$-\frac{1}{7}$	$+7$
5	3	$-\frac{1}{3}$
6	$-\frac{5}{8}$	$+\frac{8}{5}$

Finding Equations of Parallel and Perpendicular Lines

Example Find lines parallel and perpendicular to $y = 3x - 7$

Parallel $y = 3x - 5$

Perpendicular $y = -\frac{1}{3}x - 2$



Exercise

	Line	Parallel Line	Perpendicular Line
1)	$y = -\frac{1}{2}x + 5$	$y = -\frac{1}{2}x - 3$	$y = 2x + 4$
2)	$y = 4x - 3$	$y = 4x + 1$	$y = -\frac{1}{4}x - 5$
3)	$y = x$	$y = x + 2$	$y = -x - 8$
4)	$y = -\frac{2}{3}x + 1$	$y = -\frac{2}{3}x + \frac{1}{3}$	$y = \frac{3}{2}x + 5$
5)	$y = \frac{8}{5}x - \frac{1}{2}$	$y = \frac{8}{5}x + 1$	$y = -\frac{5}{8}x + 3$

<u>Example</u> Find the eqn of line perpendicular to $y = 2x + 3$ passing through $(6, 8)$

$\perp$ line $y = -\frac{1}{2}x + c$

sub $(6,8)$ $8 = -\frac{1}{2}(6) + c$ $y = -\frac{1}{2}x + 11$

$8 = -3 + c$

$8 + 3 = c$

$11 = c$

Also find the equ of a parallel line through (6,8)

// line $\quad y = 2x + c$

sub (6,8)

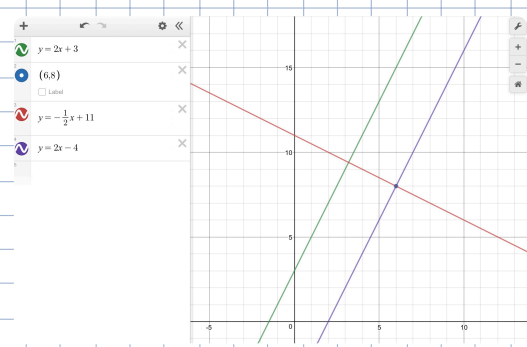
$8 = 2(6) + c$

$8 = 12 + c$

$8 - 12 = c$

$-4 = c$

$y = 2x - 4$



Exercise

Find perpendicular lines to given lines passing through given points

1) $y = -\frac{1}{3}x + 1$ through $(2, 4)$

⊥ line $\quad y = 3x + c$

sub (2,4)

$4 = 3(2) + c$

$4 = 6 + c$

$4 - 6 = c$

$-2 = c$

$y = 3x - 2$

2) $y = -5x + 3$ through $(10, 7)$

⊥ line $\quad y = \frac{1}{5}x + c$

sub $\quad 7 = \frac{1}{5}(10) + c$

$y = \frac{1}{5}x + 5$

$(10,7)$

$7 = 2 + c$

$7 - 2 = c$

$5 = c$

3) $y = \frac{2}{3}x + 7$ through $(8, 1)$

$\perp$ line $y = -\frac{3}{2}x + c$

sub $(8,1)$

$1 = -\frac{3}{2}(8) + c$

$1 = -12 + c$

$1 + 12 = c$

$13 = c$

$y = -\frac{3}{2}x + 13$

Homework

4) $y = \frac{1}{4}x - 3$ through $(3, 3)$

5) $y = -2x + 5$ through $(10, 1)$