Algebraic Proof

In the work that follows we will often need the expansion of $(a+b)^2$ and $(a-b)^2$

$$(a+b)^2 = (a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + b^2 + 2ab$$

$$(a-b)^2 = (a-b)(a-b)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 + b^2 - 2ab$$

We remember this as

"The first one squared plus the second one squared plus twice the product"

Note the product is negative in $(a-b)^2$

Examples

1) $(2x+3y)^2 = 4x^2 + 9y^2 + 12xy$

2) $(5x-2)^2 = 25x^2 + 4 - 20x$

3) $(3n+1)^2 = 9n^2 + 1 + 6n$

4) $(2n-1)^2 = 4n^2 + 1 - 4n$

Exercise

5) $(p+2q)^2 = p^2 + 4q^2 + 4pq$

6) $(3h+4k)^2 = 9h^2 + 16k^2 + 24hk$

7) $(5n-2)^2 = 25n^2 + 4 - 20n$

8) $(3n+2m)^2 = 9n^2 + 4m^2 + 12nm$

9) $(7k-5)^2 = 49k^2 + 25 - 70k$

10) $(4n+1)^2 = 16n^2 + 1 + 8n$

Algebraic Proofs GCSE Higher Tier A/A*Grades KS4 with Answers/Solutions

1. Prove that $(n+4)^2 - (3n+4) = (n+1)(n+4) + 8$
2. Prove that $(n+4)^2 - (3n+4) = (n+2)(n+3) + 6$
3. Prove that $(n+3)^2 - (3n+5) = (n+1)(n+2) + 2$
4. Prove that $(n-5)^2 - (2n-1) = (n-3)(n-9) - 1$
5. Prove that $(n-3)^2 - (n-5) = (n-3)(n-4) + 2$
6. Prove that $\frac{1}{2}(n+1)(n+2) - \frac{1}{2}n(n+1) = n+1$
7. Prove that $\frac{1}{4}(2n+1)(n+4) - \frac{1}{4}n(2n+1) = 2n+1$

8. Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of 6 for all positive integer values of n.
9. Prove that $(4n+1)^2 - (4n-1)^2$ is a multiple of 8 for all positive integer values of n.
10. Prove that $(5n+1)^2 - (5n-1)^2$ is a multiple of 5 for all positive integer values of n.

1) $(n+4)^2-(3n+4)$

$= n^2+16+8n-3n-4 \quad = n^2+5n+12$ ✓

$(n+1)(n+4)+8$

$= n^2+n+4n+4+8 \quad = n^2+5n+12$ ✓

$\therefore (n+4)^2-(3n+4) = (n+1)(n+4)+8$

---

8)

8. Prove that $(3n+1)^2-(3n-1)^2$ is a multiple of 6 for all positive integer values of n.

$(3n+1)^2-(3n-1)^2$

$= 9n^2+1+6n-[9n^2+1-6n]$

$= \cancel{9n^2}+\cancel{1}+6n-\cancel{9n^2}-\cancel{1}+6n$

$= 12n$

$= 6(2n)$ a multiple of 6

---

2. Prove that $(n+4)^2-(3n+4)=(n+2)(n+3)+6$

$(n+4)^2-(3n+4)$

$= n^2+16+8n-3n-4 \quad = n^2+5n+12$ ✓

$(n+2)(n+3)+6$

$= n^2+2n+3n+6+6 \quad = n^2+5n+12$ ✓

$\therefore (n+4)^2-(3n+4) = (n+2)(n+3)+6$

3. Prove that $(n+3)^2 - (3n+5) = (n+1)(n+2) + 2$

$(n+3)^2 - (3n+5)$

$= n^2 + 9 + 6n - 3n - 5 \quad = n^2 + 3n + 4$ ✓

$(n+1)(n+2) + 2$

$= n^2 + n + 2n + 2 + 2 \quad = n^2 + 3n + 4$ ✓

$\therefore (n+3)^2 - (3n+5) = (n+1)(n+2) + 2$