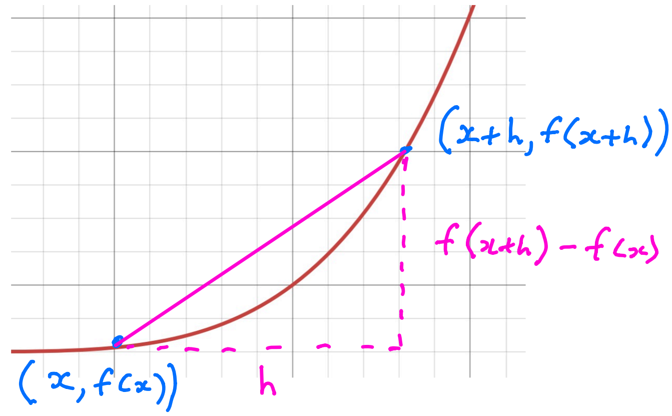
Differentiation From First Principles Exercise

The derivative of $f(x)$ written as $f'(x)$ is defined to be:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Exercise

Differentiate from first principles:

1) $f(x) = 6x^5$

2) $f(x) = x^2 + 3x + 5$

1) Let $f(x) = 6x^5$

then $f(x+h) = 6(x+h)^5$

$$= 6\left[x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5\right]$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{\cancel{6x^5} + 30x^4h + 60x^3h^2 + 60x^2h^3 + 30xh^4 + 6h^5 - \cancel{6x^5}}{h}\right]$$

$$= \lim_{h \to 0} \left[30x^4 + 60x^3h + 60x^2h^2 + 30xh^3 + 6h^4\right]$$

$$= 30x^4 + 0 + 0 + 0 + 0$$

$$\underline{f'(x) = 30x^4}$$

2) $f(x) = x^2 + 3x + 5$

$f(x+h) = (x+h)^2 + 3(x+h) + 5$

$$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$f'(x) = \lim_{h \to 0} \left[\frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h + \cancel{5} - \cancel{x^2} - \cancel{3x} - \cancel{5}}{h} \right]$$

$$f'(x) = \lim_{h \to 0} \left[2x + h + 3 \right]$$

$$f'(x) = 2x + 0 + 3$$

$$\underline{f'(x) = 2x + 3}$$