Leave blank

5.

$$I_n = \int_0^5 \frac{x^n}{\sqrt{(25 - x^2)}}\,\mathrm{d}x\,, \qquad n \geqslant 0$$

(a) Find an expression for $\displaystyle\int \frac{x}{\sqrt{(25 - x^2)}}\,\mathrm{d}x$, $0 \leqslant x \leqslant 5$.

(2)

M 3 5 1 4 5 A 0 1 2 2 8

Leave blank

2. Use calculus to find the exact value of $\int_{-2}^{1} \frac{1}{x^2+4x+13}\,\mathrm{d}x$.

(5)

Leave blank

2. (a) Given that $y = x \arcsin x, \quad 0 \leqslant x \leqslant 1$, find

(i) an expression for $\frac{dy}{dx}$,

(ii) the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$.

(3)

(b) Given that $y = \arctan(3e^{2x})$, show that

$$\frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$$

(5)

P 3 5 4 1 4 A 0 4 2 8

Leave blank

3. Show that

(a) $\displaystyle\int_5^8 \frac{1}{x^2 - 10x + 34}\,\mathrm{d}x = k\pi$, giving the value of the fraction k, **(5)**

(b) $\displaystyle\int_5^8 \frac{1}{\sqrt{(x^2 - 10x + 34)}}\,\mathrm{d}x = \ln(A + \sqrt{n})$, giving the values of the integers A and n. **(4)**

P 3 5 4 1 4 A 0 6 2 8

Leave blank

2. (a) Find

$$\int \frac{1}{\sqrt{(4x^2 + 9)}}\,dx$$

(2)

(b) Use your answer to part (a) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2 + 9)}}\,dx$$

giving your answer in the form $k \ln(a + b\sqrt{5})$, where a and b are integers and k is a constant.

(3)