E/P **2** A car manufacturer uses a model to predict the fuel consumption, y miles per gallon (mpg), for a specific model of car travelling at a speed of x mph.

$$y = -0.01x^2 + 0.975x + 16, \; x > 0$$

a Use the model to find two speeds at which the car has a fuel consumption of 32.5 mpg. **(3 marks)**

b Rewrite y in the form $A - B(x - C)^2$, where A, B and C are constants to be found. **(3 marks)**

c Using your answer to part **b**, find the speed at which the car has the greatest fuel efficiency. **(1 mark)**

d Use the model to calculate the fuel consumption of a car travelling at 120 mph. Comment on the validity of using this model for very high speeds. **(2 marks)**

a) $32.5 = -0.01x^2 + 0.975x + 16$

$0.01x^2 - 0.975x + 16.5 = 0$

By calc $x = 75.7$

$x = 21.8$

Speeds are 75.7 mph and 21.8 mph

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b) $y = -0.01x^2 + 0.975x + 16$

$y = -0.01\left[x^2 - 97.5x - 1600\right]$

$y = -0.01\left[(x - 48.75)^2 - 1600 - 2376.5625\right]$

$y = -0.01(x - 48.75)^2 + 39.765625$

$y = 39.77 - 0.01(x - 48.75)^2$

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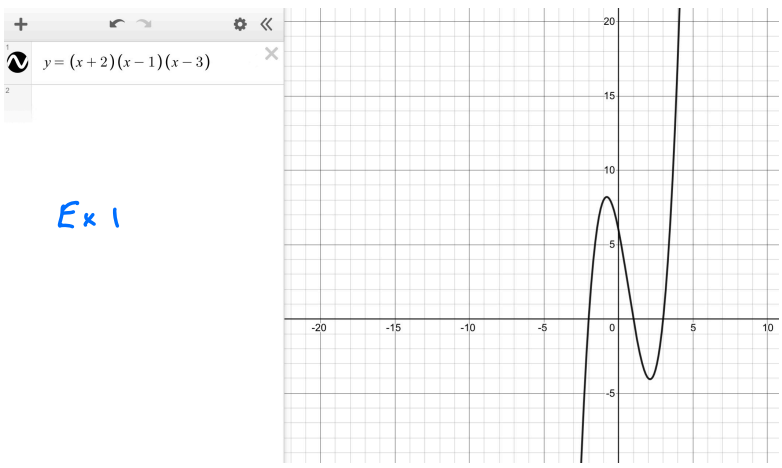
c) Max economy when speed $x = 48.75$

d) $y = 39.77 - 0.01(120 - 48.75)^2$

$y = -10.9956 < 0$

Not valid as miles per gallon is -ve

Sketching Graphs



Ex 1

$y = (x+2)(x-1)(x-3)$

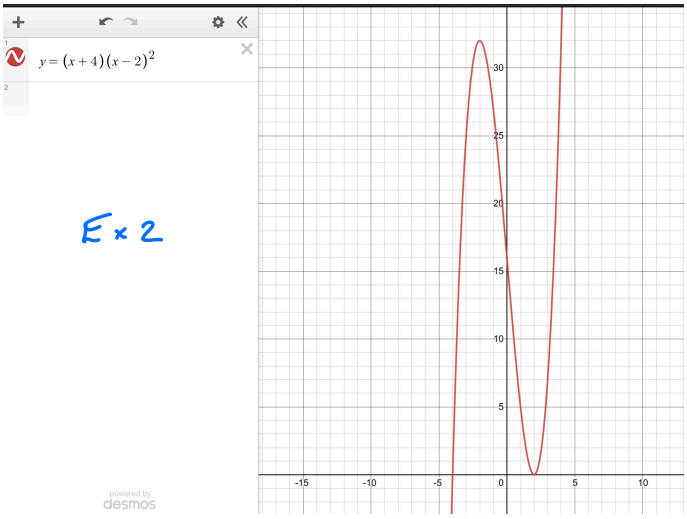
Points to note:

Cubic with positive x^3 term so standard cubic shape

Cuts x-axis when each bracket = 0

so when $x = -2, 1, 3$

Cuts y-axis at $(+2)(-1)(-3) = +6$



Ex 2

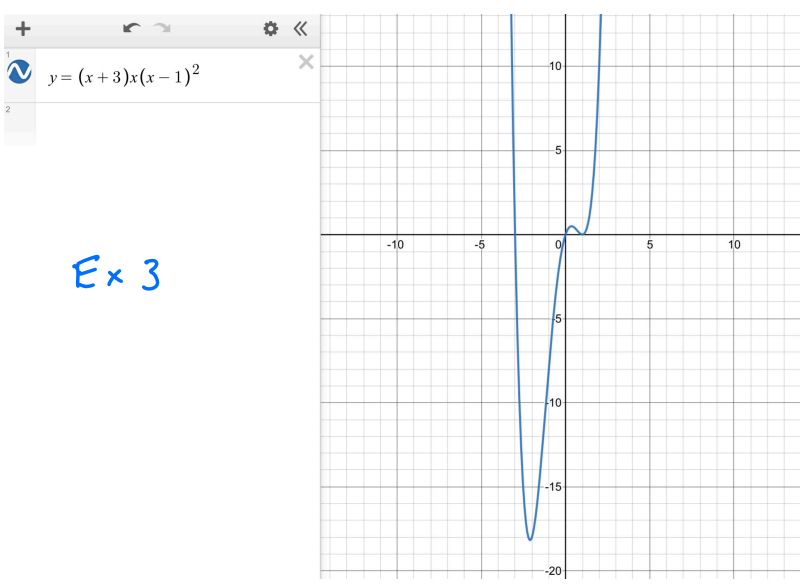
$y = (x+4)(x-2)^2$

Double root when $x = 2$

Cuts x-axis at $x = -4$

but just touches x-axis at double root $x = 2$

Cuts y-axis at $(+4)(-2)^2 = +16$



Ex 3

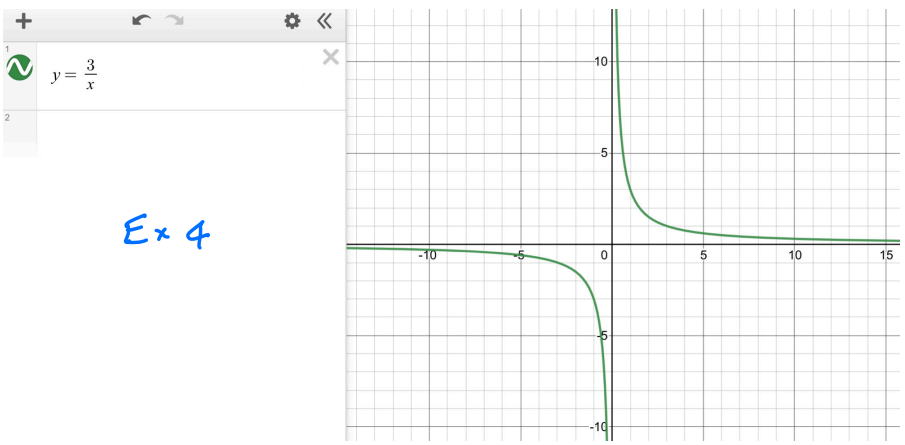
$y = (x+3)x(x-1)^2$

Quartic with positive x^4 term so W shape

Cuts x-axis at -3 and 0

Touches x-axis at $x = 1$

Cuts y-axis at $(+3)0(-1)^2 = 0$



Ex 4

$y = \frac{3}{x}$ reciprocal function

horizontal asymptote $y = 0$

vertical asymptote $x = 0$

Homework

Exercise 2H Pages 34–35 Q4

Mixed Exercise 2 Page 36 Q14

Exercise 4B Page 65 Choose a few to sketch and check with Desmos