Binomial Expansion Revision

BINOMIAL EXPANSIONS 2008-10 January 2008

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3. (a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.
(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.
(3)

a) $1 + 10\left(\frac{x}{2}\right) + \binom{10}{2}\left(\frac{x}{2}\right)^2 + \binom{10}{3}\left(\frac{x}{2}\right)^3 + \ldots$

$= 1 + 5x + \frac{10.9}{1.2}\left(\frac{x^2}{4}\right) + \frac{\overset{5}{\cancel{10}}.\overset{3}{\cancel{9}}.\cancel{8}}{1.\cancel{2}.\cancel{3}}\left(\frac{x^3}{\cancel{8}}\right) + \ldots$

$= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \ldots$

b) $x = 0.01$

$1 + 5(0.01) + \frac{45}{4}(0.01)^2 + 15(0.01)^3$

$= 1.05114$

June 2008

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3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form.
(4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2,

(b) find the value of a.
(2)

nCr

a) $1 + 10ax + \binom{10}{2}(ax)^2 + \binom{10}{3}(ax)^3 + \dots$

$= 1 + 10ax + 45a^2x^2 + 120a^3x^3 + \dots$

b) $120a^3 = 2 \times 45a^2$

$$\frac{a^3}{a^2} = \frac{90}{120}$$

$a = 0.75$

January 2009

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1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3-2x)^5$, giving each term in its simplest form.

(4)

$(a+b)^5 = a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 + b^5$

$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(3+(-2x))^5$

$= 3^5 + 5(3)^4(-2x) + 10(3)^3(-2x)^2 + 10(3)^2(-2x)^3 + 5(3)(-2x)^4 + (-2x)^5$

$= 243 - 810x + 1080x^2 - 720x^3 + 240x^4 - 32x^5$

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2. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2+kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(b) find the value of k.

(2)

a) $2^7 + 7(2)^6(kx) + \binom{7}{2}(2)^5(kx)^2 + \ldots$

$= 128 + 448kx + 672k^2x^2$

b) $672k^2 = 6 \times 448k$

$$\frac{k^2}{k} = \frac{6 \times 448}{672}$$

$$\underline{k = 4}$$

5.

$$\mathrm{f}(x) = x^3 + 3x^2 - 4x - 12$$

(a) Using the factor theorem, explain why $\mathrm{f}(x)$ is divisible by $(x+3)$.

(2)

(b) Hence fully factorise $\mathrm{f}(x)$.

(3)

(c) Show that $\dfrac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x}$ can be written in the form $A + \dfrac{B}{x}$ where A and B are integers to be found.

(3)

a) $$f(-3) = (-3)^3 + 3(-3)^2 - 4(-3) - 12$$
$$= -27 + 27 + 12 - 12$$
$$= 0$$

By factor theorem $(x+3)$ is a factor of $f(x)$

b)
$$
\begin{array}{r|llll}
 & x^2 & & -4 & \\
\hline
x+3 & x^3 & + 3x^2 & -4x & -12 \\
 & x^3 & + 3x^2 & & \\
\cline{2-3}
 & & 0 & -4x & -12 \\
 & & & -4x & -12 \\
\cline{4-5}
 & & & & 0
\end{array}
$$

$$f(x) = (x+3)(x^2-4)$$
$$= (x+3)(x+2)(x-2)$$

c)
$$\frac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x}$$

$$= \frac{(x+3)(x+2)(x-2)}{x(x^2+5x+6)}$$

$$= \frac{\cancel{(x+3)}\cancel{(x+2)}(x-2)}{x\cancel{(x+2)}\cancel{(x+3)}} = \frac{x-2}{x}$$

$$= 1 - \frac{2}{x}$$

Expand $(1+2x)^3(1+3x)^4$ upto the term in x^3

$$(1+2x)^3 = 1 + 3(2x) + 3(2x)^2 + (2x)^3$$
$$= 1 + 6x + 12x^2 + 8x^3$$

$$(1+3x)^4 = 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$$
$$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$$

Terms upto x^3

$$(1+6x+12x^2+8x^3)(1+12x+54x^2+108x^3)$$

$$\begin{array}{llll}
1 + 6x & + 12x^2 & + 8x^3 \\
 + 12x & + 72x^2 & + 144x^3 \\
 & + 54x^2 & + 324x^3 \\
 & & + 108x^3 \\
\hline
1 + 18x & + 138x^2 & + 584x^3
\end{array}$$

Laws of Logs

$$\log_n(a \times b) = \log_n a + \log_n b$$
$$\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$$
$$\log_n a^b = b \log_n a$$