

## Integration - Equations of Curves / Definite Integration

### Exercise 13C

1b)  $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$  through point  $(1, 4)$

$$\frac{dy}{dx} = 4x^3 + 2x^{-3} + 3$$

$$y = \frac{4x^4}{4} + \frac{2x^{-2}}{-2} + 3x + C$$

$$y = x^4 - \frac{1}{x^2} + 3x + C$$

Sub  $(1, 4)$   $4 = 1^4 - \frac{1}{1^2} + 3(1) + C$

$$4 = 1 - 1 + 3 + C$$

$$4 - 3 = C$$

$$1 = C$$

$$\underline{y = x^4 - \frac{1}{x^2} + 3x + 1}$$

1f)  $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}}$  through  $(0, 1)$

$$\frac{dy}{dx} = x^{3/2} + 3x^{-\frac{1}{2}}$$

$$y = \frac{x^{5/2}}{\frac{5}{2}} + \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$y = \frac{2}{5}x^{5/2} + 6x^{\frac{1}{2}} + C$$

$$\begin{array}{l} \text{Sub} \\ (0,1) \end{array} \quad \begin{aligned} 1 &= 0 + 0 + c \\ y &= \frac{2}{5}x^{5/2} + 6x^{1/2} + 1 \end{aligned}$$


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### Definite Integration

Exercise 13 D

$$\begin{aligned} 1d) \quad \int_1^3 \frac{3}{x^2} dx &= \int_1^3 3x^{-2} dx \\ &= \left[ \frac{3x^{-1}}{-1} \right]_1^3 \\ &= \left[ -\frac{3}{x} \right]_1^3 \\ &= \left( -\frac{3}{3} \right) - \left( -\frac{3}{1} \right) \\ &= -1 + 3 \\ &= 2 \end{aligned}$$


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$$\begin{aligned} 2d) \quad \int_1^8 (x^{-1/3} + 2x - 1) dx &= \left[ \frac{x^{2/3}}{\frac{2}{3}} + \frac{2x^2}{2} - x \right]_1^8 \\ &= \left[ \frac{3}{2}x^{2/3} + x^2 - x \right]_1^8 \\ &= \left( \frac{3}{2}(4) + 64 - 8 \right) - \left( \frac{3}{2} + 1 - 1 \right) \\ &= 62 - \frac{3}{2} = \underline{60.5} \end{aligned}$$

$$\begin{aligned}
 3d) \quad \int_1^4 \frac{2+\sqrt{x}}{x^2} dx &= \int_1^4 \left( \frac{2}{x^2} + \frac{\sqrt{x}}{x^2} \right) dx \\
 &= \int_1^4 \left( 2x^{-2} + x^{-\frac{1}{2}} \right) dx \\
 &= \left[ \frac{2x^{-1}}{-1} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_1^4 \\
 &= \left[ -\frac{2}{x} - \frac{2}{\sqrt{x}} \right]_1^4 \\
 &= \left( -\frac{2}{4} - \frac{2}{\sqrt{4}} \right) - \left( -\frac{2}{1} - \frac{2}{\sqrt{1}} \right) \\
 &= \left( -\frac{1}{2} - 1 \right) - \left( -2 - 2 \right) \\
 &= -\frac{1}{2} - 1 + 2 + 2 \\
 &= \frac{5}{2}
 \end{aligned}$$

Homework    Exercise 13D    (for tomorrow)

Finish    Q1 a,b,c

Q2 a,b,c

Q3 a,b,c

Q5

Q6

Q8