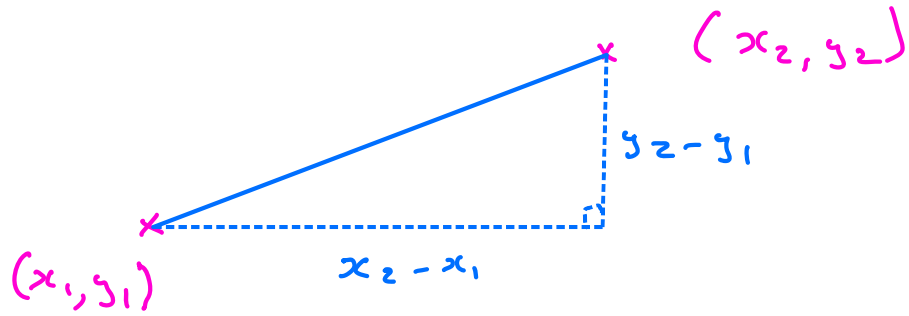
Equation of Line Through 2 Given Points

Given (x_1, y_1) and (x_2, y_2)



gradient of a line joining (x_1, y_1) and (x_2, y_2)

is given by $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples Find the gradients of lines between:

1) $\underset{x_1, y_1}{(4, 7)}$ and $\underset{x_2, y_2}{(7, 13)}$ $$m = \frac{13-7}{7-4} = \frac{6}{3} = 2$$

If we said $(4,7)$ is (x_2, y_2)
and $(7,13)$ is (x_1, y_1) $$m = \frac{7-13}{4-7} = \frac{-6}{-3} = 2$$

$\therefore$ the same answer

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2) $(-9, 1)$ and $(-3, -7)$ $$m = \frac{-7-1}{-3--9} = \frac{-8}{-3+9}$$
$$= -\frac{8}{6}$$
$$= -\frac{4}{3}$$

3) $(8,-2)$ and $(-4,5)$ $\quad m = \frac{5--2}{-4-8}$

$m = \frac{5+2}{-12} = \frac{7}{-12} = -\frac{7}{12}$

Exercise Find gradients of line segments between

1) $(5,3)$ and $(9,5)$ $\quad m = \frac{5-3}{9-5} = \frac{2}{4} = \frac{1}{2}$

2) $(4,1)$ and $(8,-3)$ $\quad m = \frac{-3-1}{8-4} = \frac{-4}{4} = -1$

3) $(6,0)$ and $(0,5)$ $\quad m = \frac{5-0}{0-6} = \frac{5}{-6} = -\frac{5}{6}$

4) $(-9,-2)$ and $(-1,-3)$ $\quad m = \frac{-3--2}{-1--9} = \frac{-3+2}{-1+9} = \frac{-1}{8} = -\frac{1}{8}$

5) $(6,-3)$ and $(2,-4)$ $\quad m = \frac{-4--3}{2-6} = \frac{-4+3}{-4} = \frac{-1}{-4} = \frac{1}{4}$

6) $(2,2)$ and $(7,-7)$ $\quad m = \frac{-7-2}{7-2} = -\frac{9}{5} = -\frac{9}{5}$

7) $(1,-3)$ and $(-3,1)$ $\quad m = \frac{1--3}{-3-1} = \frac{1+3}{-4} = \frac{4}{-4} = -1$

8) $(4,-4)$ and $(7,-4)$ $\quad m = \frac{-4--4}{7-4} = \frac{-4+4}{3} = \frac{0}{3} = 0$

9) $(8,7)$ and $(8,2)$ $\quad m = \frac{2-7}{8-8} = -\frac{5}{0}$ undefined (vertical)

10) $(-1,1)$ and $(-4,-4)$ $\quad m = \frac{-4-1}{-4--1} = \frac{-5}{-4+1} = \frac{-5}{-3} = \frac{5}{3}$

Find eqn of line through 2 given points

Example 1 Find eqn of line through $(1, 4)$ and $(4, 10)$

First find gradient $m = \frac{10-4}{4-1} = \frac{6}{3} = 2$

Line of form $y = 2x + c$

Sub $(1, 4)$

$$4 = 2(1) + c$$
$$4 = 2 + c$$
$$4 - 2 = c$$
$$2 = c$$

$$\underline{y = 2x + 2}$$

Ex 2 Find eqn of line through $(-1, 5)$ and $(3, -7)$

$$m = \frac{-7-5}{3--1} = \frac{-12}{3+1} = -\frac{12}{4} = -3$$

Line of form $y = -3x + c$

Sub for $(-1, 5)$

$$5 = -3(-1) + c$$
$$5 = 3 + c$$
$$5 - 3 = c$$
$$2 = c$$

$$\underline{y = -3x + 2}$$

Exercise Find eqns of lines through

1) $(5, 1)$ and $(1, 4)$ $m = \frac{4-1}{1-5} = \frac{3}{-4} = -\frac{3}{4}$

line of form $y = -\frac{3}{4}x + c$

Sub $(1,4)$

$$4 = -\frac{3}{4}(1) + c$$
$$4 = -\frac{3}{4} + c$$
$$\frac{16}{4} + \frac{3}{4} = c$$
$$\frac{19}{4} = c$$

$$y = -\frac{3}{4}x + \frac{19}{4}$$

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2) $(-2,4)$ and $(1,16)$

$$m = \frac{16-4}{1--2} = \frac{12}{3} = 4$$

$$y = 4x + c$$

Sub $(1,16)$

$$16 = 4(1) + c$$
$$16-4 = c$$
$$12 = c$$

$$\underline{y = 4x + 12}$$

3) $(5,7)$ and $(7,3)$

$$m = \frac{3-7}{7-5} = \frac{-4}{2} = -2$$

$$y = -2x + c$$

Sub $(5,7)$

$$7 = -2(5) + c$$
$$7 = -10 + c$$
$$17 = c$$

$$\underline{y = -2x + 17}$$

4) $(5,10)$ and $(10,5)$

$$m = \frac{5-10}{10-5} = \frac{-5}{5} = -1$$

$$y = -x + c$$

Sub $(5,10)$

$$10 = -5 + c$$
$$15 = c$$

$$\underline{y = -x + 15}$$

5) $(-2,3)$ and $(4,6)$

$$m = \frac{6-3}{4--2} = \frac{3}{6} = \frac{1}{2}$$

$$y = \frac{1}{2}x + c$$

Sub $(4,6)$

$$6 = \frac{1}{2}(4) + c$$
$$6 = 2 + c$$
$$4 = c$$

$$\underline{y = \frac{1}{2}x + 4}$$