Recurring Decimals to Fractions

Ex1   Convert $0.4\dot{7}$ into a fraction

$$\text{Let } x = 0.4777$$
$$10x = 4.77\dot{7} \quad ①$$
$$100x = 47.77\dot{7} \quad ②$$

② - ①
$$90x = 43$$
$$x = \frac{43}{90}$$

Ex2

Convert $0.6\dot{1}\dot{3}$ into a fraction

$$\text{Let } x = 0.6131313$$
$$10x = 6.131313 \quad ①$$
$$1000x = 613.131313 \quad ②$$

② - ①
$$990x = 607$$
$$x = \frac{607}{990}$$

Ex3   Convert $0.\dot{4}3\dot{2}$ into a fraction

$$\text{Let } x = 0.432432 \quad ①$$
$$1000x = 432.432432 \quad ②$$

② −① $\quad 999x = 432$

$$x = \frac{432}{999} = \frac{144}{333} = \frac{48}{111} = \frac{16}{37}$$

Exercise Convert to fractions

1) $\quad 0.5\dot{7}$

Let $\quad x = 0.5777\dot{7}$
$\quad 10x = 5.777\dot{7}$
$\quad 100x = 57.777\dot{7}$
$\quad 90x = 52$
$\quad x = \frac{52}{90} = \frac{26}{45}$

2) $\quad 0.3\dot{5}\dot{1}$

Let $\quad x = 0.351\dot{5}\dot{1}$
$\quad 10x = 3.51\dot{5}\dot{1}$
$\quad 1000x = 351.51\dot{5}\dot{1}$
$\quad 990x = 348$
$\quad x = \frac{348}{990} = \frac{58}{165}$

3) $\quad 0.\dot{2}8\dot{9}$

Let $\quad x = 0.289289\dot{}$
$\quad 1000x = 289.289289\dot{}$

$$999x = 289$$

$$x = \frac{289}{999}$$

$$999x = 289$$

$$x = \frac{289}{999}$$