

## Geometric Distribution

$$P(X=x) = p(1-p)^{x-1} \quad x = 1, 2, 3, \dots$$

where  $p$  is the constant probability of success in each individual trial.

If  $X \sim Geo(p)$  Cumulative distribution

$$P(X \leq x) = 1 - (1-p)^x$$

If  $X \sim Geo(p)$

$$\text{Mean } X = E(X) = \mu = \frac{1}{p}$$

$$\text{Variance} = \text{Var}(X) = \sigma^2 = \frac{1-p}{p^2}$$

### Exercise 3A

- $x \sim Geo(0.15)$
- $P(X=10) = 0.85^9 \times 0.15$
  - $P(X \leq 7) = 1 - P(X > 7)$   
 $= 1 - 0.85^7$
  - $P(3 \leq X \leq 12) = P(X \leq 12) - P(X \leq 2)$   
 $= 1 - 0.85^{12} - (1 - 0.85^2)$

s) Roll 4 to start 4 sided dice

$$a) i) P(X=1) = \frac{1}{4}$$

$$\text{ii) } P(X=5) = \left(\frac{1}{4}\right)^4 \times \frac{1}{4}$$

$$\begin{aligned}\text{iii) } P(X \leq 4) &= 1 - P(X > 4) \\ &= 1 - \left(\frac{3}{4}\right)^4\end{aligned}$$

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$$7) X \sim Geom(0.032)$$

$$\text{a) } P(X=x) = 0.0203$$

$$(0.968)^{x-1} \times 0.032 = 0.0203$$

$$0.968^{x-1} = \frac{0.0203}{0.032}$$

$$(x-1)\ln(0.968) = \ln\left(\frac{0.0203}{0.032}\right)$$

$$x-1 = \frac{\ln\left(\frac{0.0203}{0.032}\right)}{\ln(0.968)}$$

$$x-1 = 13.99$$

$$x = 14.99$$

$$\underline{x=15}$$

$$\text{b) } P(X \leq x) < 0.1$$

$$1 - (1-p)^x < 0.1$$

$$1 - 0.968^x < 0.1$$

$$1 - 0.1 < 0.968^x$$

$$0.9 < 0.968^x$$

True for  $x=3$  but not for  $x=4$

$$\text{so } x=3$$

$$c) P(X \geq x_c) < 0.05$$

$$P(X > x-1) < 0.05$$
$$0.968^{(x-1)} < 0.05$$

$$(x-1) \ln 0.968 < \ln(0.05)$$

$$(x-1) > \frac{\ln(0.05)}{\ln(0.968)}$$

$$x-1 > 92.11$$

$$x > 93.11$$

$$\text{Min } \underline{x = 94}$$

$$9) X \sim \text{Geo}(0.05)$$

$$a) P(X=10) = 0.95^9 \times 0.05$$

$$b) P(X \geq 15) = P(X > 14) = 0.95^{14}$$

$$11) X \sim \text{Poisson}(4)$$

$$P(X \geq 5) = 1 - P(X \leq 4)$$
$$= 1 - 0.6288369454$$

$$= 0.3711630546$$

$$P(Y=5) = 0.6288369^4 \times 0.3712 = 0.0580$$

$$P(Y>8) = 0.6288369^8 = 0.0245$$

## Exercise 305

7) a) Geometric Dist

b) Prob constant and independent trials

c)  $P(X=2) = 0.16 \quad p < 0.5 \quad \text{Find } p$ 

$$(1-p)p = 0.16$$

$$p - p^2 = 0.16 \quad 0 = p^2 - p + 0.16$$

By calc  $p = \frac{1}{5}$   ~~$p = \frac{4}{5}$~~

$$\underline{p = 0.2}$$

$$E(X) = \frac{1}{p} = \frac{1}{0.2} = 5$$

$$\text{Var}(X) = \frac{1-p}{p^2} = \frac{1 - \frac{1}{5}}{\left(\frac{1}{5}\right)^2} = \frac{4}{5} \times 25 \\ = 20$$