

8. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x .

- (a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

- (b) Hence find the set of possible values of k .

(4)

a) no real solutions $\Rightarrow b^2 - 4ac < 0$

$$x^2 + kx + 8 - k = 0$$

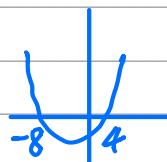
$$k^2 - 4 \times 1 \times (8 - k) < 0$$

$$k^2 - 32 + 4k < 0$$

$$k^2 + 4k - 32 < 0$$

b)

$$(k - 4)(k + 8) < 0$$



$$-8 < k < 4$$



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8. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q .

(3)

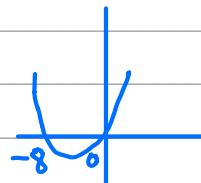
a) no real roots $\Rightarrow b^2 - 4ac < 0$

$$2qx^2 + qx - 1 = 0$$

$$q^2 - 4 \times 2q \times (-1) < 0$$

$$q^2 + 8q < 0$$

b) $q(q + 8) < 0$



$$-8 < q < 0$$



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7. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x .

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k .

(4)

a) 2 real solutions $\Rightarrow b^2 - 4ac > 0$

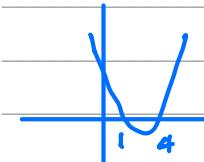
$$kx^2 + 4x + (5 - k) = 0$$

$$4^2 - 4k(5 - k) > 0$$

$$16 - 20k + 4k^2 > 0$$

$$k^2 - 5k + 4 > 0$$

b) $(k - 1)(k - 4) > 0$



Either $k < 1$ or $k > 4$



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6. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p .

(4)

$$\text{Equal roots} \Rightarrow b^2 - 4ac = 0$$

$$x^2 + 3px + p = 0$$

$$(3p)^2 - 4 \times 1 \times p = 0$$

$$9p^2 - 4p = 0$$

$$p(9p - 4) = 0$$

$$\Rightarrow p = 0 \quad \text{or} \quad p = \frac{4}{9}$$

$$\text{Given } p \neq 0 \quad \therefore p = \frac{4}{9}$$



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10.

$$f(x) = x^2 + 4kx + (3 + 11k), \quad \text{where } k \text{ is a constant.}$$

- (a) Express $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k . (3)

Given that the equation $f(x) = 0$ has no real roots,

- (b) find the set of possible values of k . (4)

Given that $k = 1$,

- (c) sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.

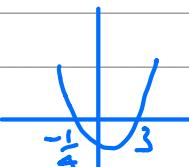
a)
$$f(x) = (x + 2k)^2 + (3 + 11k - 4k^2) \quad (3)$$

$$p = 2k, \quad q = 3 + 11k - 4k^2$$

b) no real roots $\Rightarrow 3 + 11k - 4k^2 > 0$

$$\Rightarrow 4k^2 - 11k - 3 < 0$$

$$(4k + 1)(k - 3) < 0$$

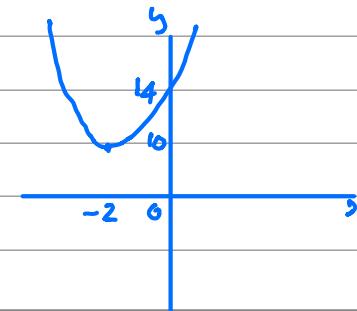


$$-\frac{1}{4} < k < 3$$

c) $k = 1 \quad f(x) = (x + 2)^2 + (3 + 11 - 4)$

$$f(x) = (x + 2)^2 + 10$$

Minimum point
(-2, 10)



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4. (a) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q$$

where p and q are integers to be found.

(2)

- (b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

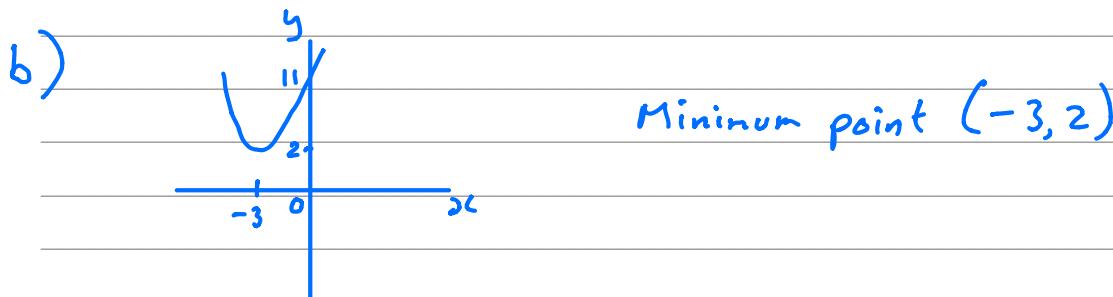
(2)

- (c) Find the value of the discriminant of $x^2 + 6x + 11$

(2)

a)
$$\begin{aligned} x^2 + 6x + 11 &= (x+3)^2 + 11 - 9 \\ &= (x+3)^2 + 2 \end{aligned}$$

$p = 3, q = 2$



c) $x^2 + 6x + 11$ Discriminant $= b^2 - 4ac$

$$\begin{aligned} &= 6^2 - 4 \times 1 \times 11 \\ &= 36 - 44 \\ &= -8 \end{aligned}$$

