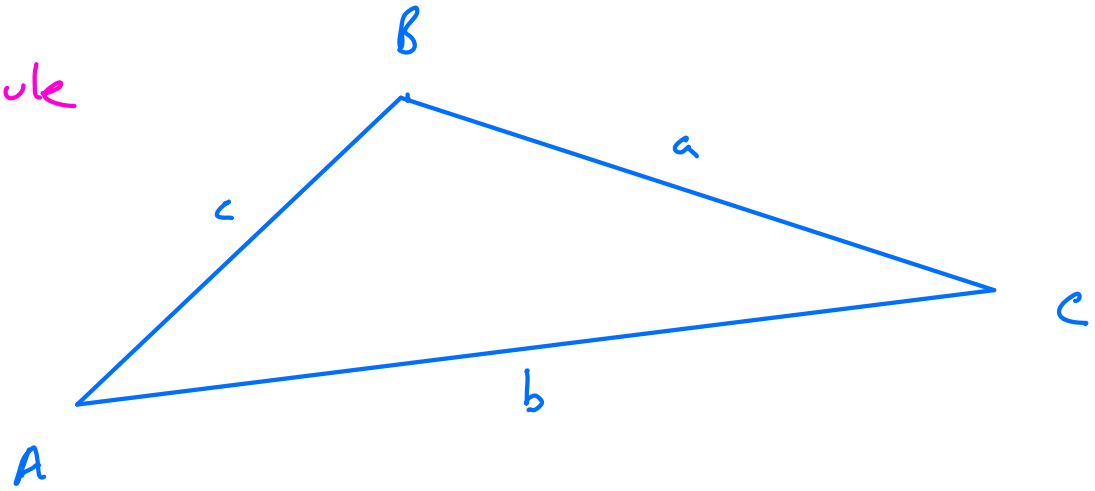
Sine Rule and Cosine Rule

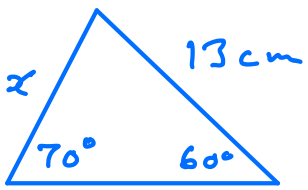
Sine Rule



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1

Find x

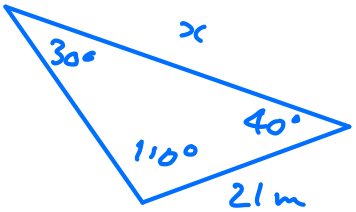


$$\frac{13}{\sin 70^\circ} = \frac{x}{\sin 60^\circ}$$

$$\frac{13}{\sin 70^\circ} \times \sin 60^\circ = x$$

$$x = 11.98 \text{ cm}$$

Ex 2



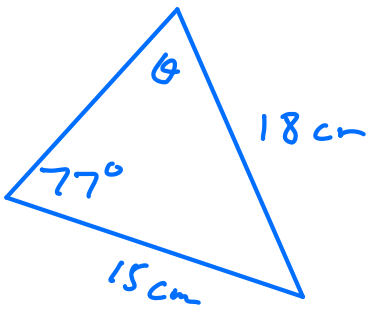
Find x

$$\frac{x}{\sin 110^\circ} = \frac{21}{\sin 30^\circ}$$

$$x = \frac{21}{\sin 30^\circ} \times \sin 110^\circ$$

$$x = 39.47 \text{ m}$$

Ex 3 Finding an angle θ



$$\frac{15}{\sin\theta} = \frac{18}{\sin 77^\circ}$$

$$\frac{\sin\theta}{15} = \frac{\sin 77^\circ}{18}$$

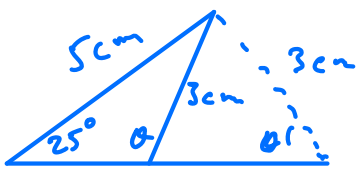
$$\sin\theta = \frac{\sin 77^\circ}{18} \times 15$$

$$\sin\theta = 0.811975$$

$$\theta = \sin^{-1}(0.811975)$$

$$\theta = 54.3^\circ$$

Ex 4 This example is called the ambiguous case of sine rule



Two different triangles can be constructed with a 25° angle adjacent to a 5cm side and opposite a 3cm side

In this case we cannot be sure whether

The angle opposite the 5 cm side is the obtuse angle θ or the acute angle θ'

Sine Rule $$\frac{3}{\sin 25^\circ} = \frac{5}{\sin\theta}$$

$$\frac{\sin 25^\circ}{3} = \frac{\sin\theta}{5}$$

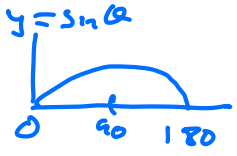
$$\frac{\sin 25^\circ}{3} \times 5 = \sin\theta$$

$$\sin^{-1}\left(\frac{\sin 25^\circ}{3} \times 5\right) = \theta$$

$$\theta = 44.8^\circ$$

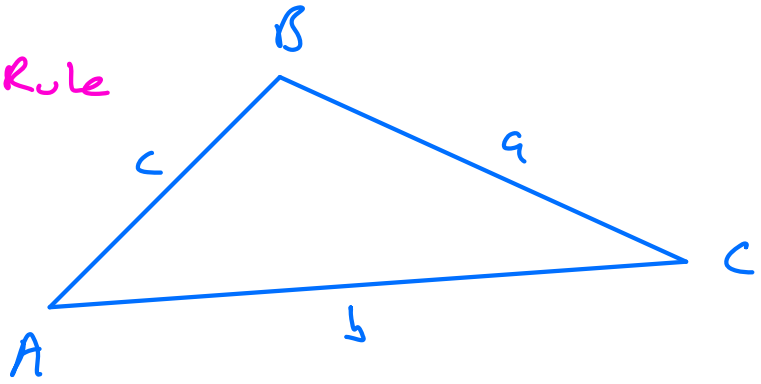
or $$\theta = 180^\circ - 44.8^\circ = 135.2^\circ$$

This is because $\sin\theta = \sin(180-\theta)$



To check whether the obtuse angle is possible we can add 135.2° to 25° to make sure the 180° sum for a triangle has been exceeded.

Cosine Rule



$$a^2 = b^2 + c^2 - 2bc\cos A$$

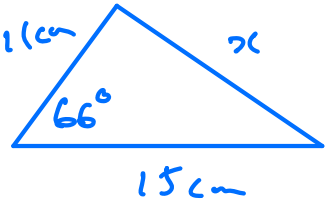
To find an angle we can rearrange this formula

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Ex 5



Find x

$$x^2 = 15^2 + 11^2 - 2\times 11\times 15\cos 66^\circ$$

$$x^2 = 211.7769$$

$$x = \sqrt{211.7769}$$

$$x = 14.55\text{ cm}$$