Factor Theorem (On Syllabus)

Remainder Theorem (No longer on syllabus)

Suppose $f(x) = g(x)(x-a) + r$

Example $\quad f(x) = x^3 + 2x^2 - 3x + 4$

$$
\begin{array}{r|l}
 & x^2 + 7x + 32 \\
\hline
x - 5 & x^3 + 2x^2 - 3x + 4 \\
 & x^3 - 5x^2 \\
 & \quad + 7x^2 - 3x \\
 & \quad + 7x^2 - 35x \\
 & \qquad + 32x + 4 \\
 & \qquad + 32x - 160 \\
 & \qquad\qquad + 164
\end{array}
$$

$$f(x) = (x^2 + 7x + 32)(x-5) + 164$$

$$f(x) = g(x)(x-a) + r$$

$$f(a) = g(a)(a-a) + r$$

$$f(a) = r$$

The remainder when $f(x)$ is divided by $(x-a)$ is simply $f(a)$

eg $\quad f(5) = 5^3 + 2(5)^2 - 3(5) + 4$

$$= 125 + 50 - 15 + 4$$
$$= 164$$

Formally,

The remainder when a polynomial $f(x)$ is divided by a linear factor $(x-a)$ is given by $f(a)$

Factor Theorem

$(x-a)$ is a factor of $f(x)$ if and only if $f(a) = 0$

Example $\quad f(x) = x^3 - 6x^2 + 11x - 6$

Factorise this function

$$f(1) = 1^3 - 6(1)^2 + 11(1) - 6 = 0 \checkmark$$

$\therefore (x-1)$ is a factor

$$f(2) = 2^3 - 6(2)^2 + 11(2) - 6 = 0 \checkmark$$

$(x-2)$ is a factor

$$f(x) = (x-2)(x-1)(x-3)$$

Check $\quad f(3) = 3^3 - 6(3)^2 + 11(3) - 6 = 0 \checkmark$

Ex2 Solve $x^3 - 5x^2 - 2x + 24 = 0$

$f(1) = 1 - 5 - 2 + 24 \neq 0$

$f(2) = 2^3 - 5(2)^2 - 2(2) + 24 = 8$

$f(3) = 3^3 - 5(3)^2 - 2(3) + 24$

$= 27 - 45 - 6 + 24 = 0$

$\therefore$ by factor theorem $(x-3)$ is a factor

$$
\begin{array}{r|l}
 & x^2 - 2x - 8 \\
\hline
x - 3 & x^3 - 5x^2 - 2x + 24 \\
 & \underline{x^3 - 3x^2} \\
 & \quad -2x^2 - 2x \\
 & \quad \underline{-2x^2 + 6x} \\
 & \qquad -8x + 24 \\
 & \qquad \underline{-8x + 24}
\end{array}
$$

$(x-3)(x^2 - 2x - 8) = 0$

$(x-3)(x-4)(x+2) = 0$

$x = 3, \quad x = 4, \quad x = -2$

Ex3 $p(x) = x^3 - 6x^2 + 9x + k$

has a factor $x - 4$

i) Find k

By factor theorem $f(4)=0$

$$4^3 - 6(4)^2 + 9(4) + k = 0$$

$$64 - 96 + 36 + k = 0$$

$$k = -4$$

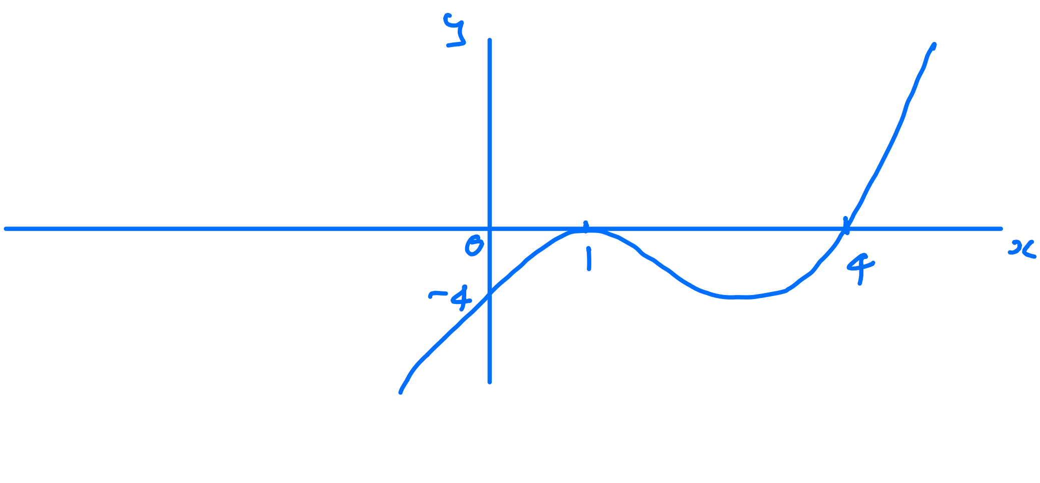
$$p(x) = x^3 - 6x^2 + 9x - 4$$

ii) Find other factors

$$p(1) = 1^3 - 6(1)^2 + 9(1) - 4 = 0$$

$$p(x) = (x-1)(x-1)(x-4)$$

iii)



Classwork and Homework

Exercise 7C Even Numbers