

### Composite Functions

$$\text{Let } f(x) = x^2, \quad g(x) = 2x+3, \quad h(x) = \frac{1}{x}$$

$$\text{Find } f(2), \quad g(3), \quad h(4)$$

$$f(2) = 2^2 = 4$$

$$g(3) = 2(3) + 3 = 9$$

$$h(4) = \frac{1}{4}$$


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$$fg(x) = f(2x+3) = (2x+3)^2 = 4x^2 + 12x + 9$$

$$gf(x) = g(x^2) = 2x^2 + 3$$

$$hf_g(x) = hf(2x+3) = h((2x+3)^2) = \frac{1}{(2x+3)^2}$$

$$\text{Find } hf_g(1) = \frac{1}{(2(1)+3)^2} = \frac{1}{5^2} = \frac{1}{25}$$


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### Exercise

$$f(x) = 2x+5$$

$$g(x) = 3x-1$$

$$h(x) = \frac{1}{x^2}$$

Find

$$1) \quad fg(x) = f(3x-1) = 2(3x-1) + 5 = 6x + 3$$

$$2) \quad gh(x) = g\left(\frac{1}{x^2}\right) = 3\left(\frac{1}{x^2}\right) - 1 = \frac{3}{x^2} - 1$$

$$3) \quad gf(x) = g(2x+5) = 3(2x+5) - 1 = 6x + 14$$

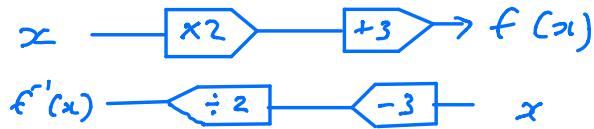
$$4) \quad hf(x) = h(2x+5) = \frac{1}{(2x+5)^2}$$

$$5) \quad fgh(x) = fg\left(\frac{1}{x^2}\right) = f\left(\frac{3}{x^2} - 1\right) = 2\left(\frac{3}{x^2} - 1\right) + 5 = \frac{6}{x^2} + 3$$


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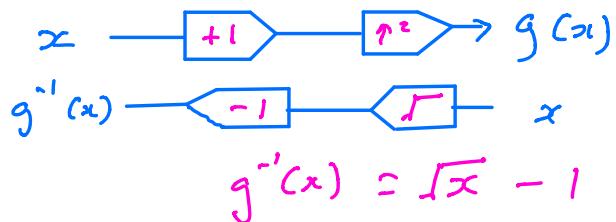
## Inverse Functions

Ex1  $f(x) = 2x + 3$



$$f^{-1}(x) = \frac{x - 3}{2}$$

Exercise Find  $g^{-1}(x)$  if  $g(x) = (x + 1)^2$



More sophisticated Method

Ex1  $f(x) = 2x + 3$

Let  $y = 2x + 3$

Swap  
variables

$$x = 2y + 3$$

Make  
 $y$  subject

$$x - 3 = 2y$$

$$\frac{x - 3}{2} = y$$

$$f^{-1}(x) = \frac{x - 3}{2}$$

$$Ex 2 \quad g(x) = (x+1)^2$$

$$\text{Let } y = (x+1)^2$$

swap  
variables

$$x = (y+1)^2$$

$$\sqrt{x} = y+1$$

$$\sqrt{x}-1 = y$$

$$g^{-1}(x) = \sqrt{x} - 1$$

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Ex 3

$$h(x) = \frac{x-2}{x+3}$$

find  $h^{-1}(x)$

$$\text{Let } y = \frac{x-2}{x+3}$$

swap  
variables

$$x = \frac{y-2}{y+3}$$

$$x(y+3) = y-2$$

$$xy + 3x = y - 2$$

$$3x + 2 = y - xy$$

$$3x + 2 = y(1-x)$$

$$\frac{3x+2}{1-x} = y$$

$$h^{-1}(x) = \frac{3x+2}{1-x}$$

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