Size and Power of Tests

		Truth	
		H_0 *is* true	H_0 *is* false
Conclusion of test	Accept H_0	OK	P(Type II error)
	Reject H_0	Size = P(Type I error)	Power = 1 − (Type II error)

Exercise 8C

1) $X \sim N(\mu, 3^2)$ $\qquad \bar{X} \sim N\left(20, \left(\frac{3}{\sqrt{25}}\right)^2\right)$

$H_0: \mu = 20$ $\qquad P(\bar{X} > 20.99) = 5\%$

$H_1: \mu > 20$

sig level 5%

a) Critical Region $\bar{X} > 20.99$

b) $\bar{Y} \sim N\left(20.8, \left(\frac{3}{\sqrt{25}}\right)^2\right)$ $\qquad P(\bar{Y} < 20.99)$

$= 0.6243$

Type II error $= 0.6243$

Power $= 1 - 0.6243 = 0.3757$

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3) $X \sim Po(4.5)$ $\qquad H_0: \lambda = 4.5$

$H_1: \lambda < 4.5$

sig level 5%

a) size of test $= 0.0111$ $\qquad P(X=0) = 0.0111$

b) $Y \sim Po(4.1)$ $\qquad P(Y > 0) = 1 - e^{-4.1}$

$= 0.9834$

Type II error

Power $= 1 - 0.9834 = 0.0166$

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5) $H_0: p = 0.3$ $\quad X \sim B(10, 0.3)$

$H_1: p > 0.3$

a) $p = 0.4$ $\quad Y \sim B(10, 0.4)$ $\quad P(Y \leq 6) = 0.9452$

$=$ Type II error

$$\text{Power} = 1 - 0.9452 = 0.0548$$

b) $p = 0.8$ $\quad Y \sim B(10, 0.8)$ $\quad P(X \leq 6) = 0.1209$

$=$ Type II error

$$\text{Power} = 1 - 0.1209 = 0.8791$$

c) Test more powerful the further p is away from 0.3

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7) $H_0: p = 0.01$ $\quad$ 5% sig level

$H_1: p > 0.01$

a) $P(X \leq x) = 1 - (1-p)^x$

Require $\quad 1 - (1 - 0.01)^x < 0.05$

$$0.95 < 0.99^x$$

$$\ln 0.95 < x \ln 0.99$$

$$\frac{\ln 0.95}{\ln 0.99} > x$$

$$5.1 > x$$

$$x \leq 5$$

b) $X \sim Geo(0.2)$

$$P(X>5) = 1 - P(X \leq 5)$$
$$= 1 - (1 - 0.8^5)$$
$$= \underline{0.3277}$$

Type II error

$$\text{Power} = 1 - 0.3277 = 0.6723$$