Completing the square

1) Write $x^2+10x+3$ in the form $(x+a)^2+b$

$= (x+5)^2+3-25$

$= (x+5)^2-22$

$(x+5)(x+5)$

$x^2+5x+5x+25$

$x^2+10x+25$

2) x^2-6x+1

$= (x-3)^2+1-9$

$= (x-3)^2-8$

3) x^2+3x+4

$= (x+\frac{3}{2})^2+4-\frac{9}{4}$

$= (x+\frac{3}{2})^2+\frac{16}{4}-\frac{9}{4}$

$= (x+\frac{3}{2})^2+\frac{7}{4}$

Exercise

1) $x^2+8x+10$

$= (x+4)^2+10-16$

$= (x+4)^2-6$

2) x^2-4x-3

$= (x-2)^2-3-4$

$= (x-2)^2-7$

3) x^2+x+4

$= (x+\frac{1}{2})^2+4-\frac{1}{4}$

$= (x+\frac{1}{2})^2+\frac{15}{4}$

4) x^2-2x+7

$= (x-1)^2+7-1$

$= (x-1)^2+6$

Expanding Brackets

Ex1 $(x+4)(x+3)$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

Ex2 $(x-3)(x+2)$

$= x^2 - 3x + 2x - 6$

$= x^2 - x - 6$

Ex3 $(2x+5)(x-4)$

$= 2x^2 + 5x - 8x - 20$

$= 2x^2 - 3x - 20$

Exercise

1) $(x+10)(x+3) = x^2 + 10x + 3x + 30$

$= x^2 + 13x + 30$

2) $(x-4)(x-2) = x^2 - 4x - 2x + 8$

$= x^2 - 6x + 8$

3) $(2x+3)(x+5) = 2x^2 + 3x + 10x + 15$

$= 2x^2 + 13x + 15$

Trinomials

$(x+2)(x+5)(x+1)$

$[x^2 + 2x + 5x + 10](x+1)$

$[x^2 + 7x + 10](x+1)$

$$= x^3 + 7x^2 + 10x$$
$$\quad + x^2 + 7x + 10$$
$$= x^3 + 8x^2 + 17x + 10$$

Ex2 $(x+3)(x-2)(x+4)$

$$[(x+3)(x-2)](x+4)$$

$$[x^2 + 3x - 2x - 6](x+4)$$

$$[x^2 + x - 6](x+4)$$

$$= x^3 + x^2 - 6x$$
$$\quad + 4x^2 + 4x - 24$$
$$= x^3 + 5x^2 - 2x - 24$$

Exercise

1) $(x+1)(x+2)(x+3)$

$$= [(x+1)(x+2)](x+3)$$
$$= [x^2 + x + 2x + 2](x+3)$$
$$= (x^2 + 3x + 2)(x+3)$$
$$= x^3 + 3x^2 + 2x$$
$$\quad + 3x^2 + 9x + 6$$
$$= x^3 + 6x^2 + 11x + 6$$

2) $(x+2)(x+3)(x+4)$

$$= [x^2 + 2x + 3x + 6](x+4)$$
$$= [x^2 + 5x + 6](x+4)$$
$$= x^3 + 5x^2 + 6x$$
$$\quad + 4x^2 + 20x + 24$$
$$= x^3 + 9x^2 + 26x + 24$$

3) $(x+1)(x+1)(x+1)$

$= [x^2 + x + x + 1](x+1)$

$= [x^2 + 2x + 1](x+1)$

$= x^3 + 2x^2 + x$
$\quad + x^2 + 2x + 1$

$= x^3 + 3x^2 + 3x + 1$

Rearranging Formulae

Ex1 $\quad 2x + py = 3y$ $\quad$ Make y subject

$$2x = 3y - py$$

$$2x = y(3-p)$$

$$\frac{2x}{3-p} = y$$

Ex2 $\quad y = \dfrac{x+3}{x-4}$ $\quad$ Make x subject

$$y(x-4) = x+3$$

$$yx - 4y = x + 3$$

$$yx - x = 3 + 4y$$

$$x(y-1) = 3 + 4y$$

$$x = \frac{3+4y}{y-1}$$

Exercise

Make q subject

$$5x - 2q = 3 + q$$

$$5x - 3 = q + 2q$$

$$5x - 3 = 3q$$

$$\frac{5x-3}{3} = q$$