

Completing the Square

$$\begin{aligned}
 \text{Consider } & (x+a)^2 \\
 &= (x+a)(x+a) \\
 &= x^2 + ax + ax + a^2 \\
 &= x^2 + 2ax + a^2
 \end{aligned}$$

When completing the square we wish to write a quadratic expression as $(x+a)^2 + b$

$$\begin{aligned}
 \text{Ex1} \quad & x^2 + 6x + 10 \\
 &= (x+3)^2 + 10 - 9 \\
 &= (x+3)^2 + 1
 \end{aligned}
 \qquad \qquad \qquad
 \begin{aligned}
 & (x+3)(x+3) \\
 &= x^2 + 3x + 3x + 9 \\
 &= x^2 + 6x + 9
 \end{aligned}$$

writing $(x+3)^2$ introduced $+9$ we did not want so we subtracted it. The $+3$ came from half the $+6$

$$\begin{aligned}
 \text{Ex2} \quad & x^2 + 8x + 5 \\
 &= (x+4)^2 + 5 - 16 \\
 &= (x+4)^2 - 11
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex3} \quad & x^2 - 10x + 50 \\
 &= (x-5)^2 + 50 - 25 \\
 &= (x-5)^2 + 25
 \end{aligned}$$

Exercise

$$\begin{aligned}1) \quad & x^2 + 2x + 3 \\&= (x+1)^2 + 3 - 1 \\&= (x+1)^2 + 2\end{aligned}$$

$$\begin{aligned}2) \quad & x^2 + 10x + 30 \\&= (x+5)^2 + 30 - 25 \\&= (x+5)^2 + 5\end{aligned}$$

$$\begin{aligned}3) \quad & x^2 - 4x + 10 \\&= (x-2)^2 + 10 - 4 \\&= (x-2)^2 + 6\end{aligned}$$

$$\begin{aligned}4) \quad & x^2 - 8x - 3 \\&= (x-4)^2 - 3 - 16 \\&= (x-4)^2 - 19\end{aligned}$$

$$\begin{aligned}5) \quad & x^2 + 5x + 10 \\&= \left(x + \frac{5}{2}\right)^2 + 10 - \frac{25}{4} \\&= \left(x + \frac{5}{2}\right)^2 + \frac{40}{4} - \frac{25}{4} \\&= \left(x + \frac{5}{2}\right)^2 + \frac{15}{4}\end{aligned}$$

$$\begin{aligned}6) \quad & x^2 - 3x + 5 \\&= \left(x - \frac{3}{2}\right)^2 + 5 - \frac{9}{4} \\&= \left(x - \frac{3}{2}\right)^2 + \frac{20}{4} - \frac{9}{4} = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}\end{aligned}$$

$$\begin{aligned}7) \quad & x^2 + x + 3 \\&= \left(x + \frac{1}{2}\right)^2 + 3 - \frac{1}{4} \\&= \left(x + \frac{1}{2}\right)^2 + \frac{12}{4} - \frac{1}{4} = \left(x + \frac{1}{2}\right)^2 + \frac{11}{4}\end{aligned}$$