Completing the Square

Ex1 $x^2 + 12x - 5$

$= (x+6)^2 - 5 - 36$

$= (x+6) - 41$

Ex2 $x^2 - 5x + 10$

$= (x - \frac{5}{2})^2 + 10 - \frac{25}{4}$

$= (x - \frac{5}{2})^2 + \frac{40}{4} - \frac{25}{4}$

$= (x - \frac{5}{2})^2 + \frac{15}{4}$

Exercise Complete the Square

1) $x^2 + 4x + 10 = (x+2)^2 + 10 - 4 = (x+2)^2 + 6$

2) $x^2 - 6x + 1 = (x-3)^2 + 1 - 9 = (x-3)^2 - 8$

3) $x^2 + 7x + 8 = (x + \frac{7}{2})^2 + 8 - \frac{49}{4} = (x + \frac{7}{2})^2 - \frac{17}{4}$

4) $x^2 - 3x - 2 = (x - \frac{3}{2})^2 - 2 - \frac{9}{4} = (x - \frac{3}{2})^2 - \frac{17}{4}$

5) $x^2 - x + 7 = (x - \frac{1}{2})^2 + 7 - \frac{1}{4} = (x - \frac{1}{2})^2 + \frac{27}{4}$

Transformations of Functions

Let $y = f(x)$

Example

$y = x^2$

$y = f(x-a)$ is a translation by $\begin{pmatrix} a \\ 0 \end{pmatrix}$

$y = (x-a)^2$

$y = f(x) + b$ is a translation by $\begin{pmatrix} 0 \\ b \end{pmatrix}$

$y = x^2 + b$

$y = f(x-a) + b$ is a translation by $\begin{pmatrix} a \\ b \end{pmatrix}$

$y = (x-a)^2 + b$

Translation means move

Translation by $\begin{pmatrix} a \\ b \end{pmatrix}$ means move a units in x-direction and b units in the y-direction

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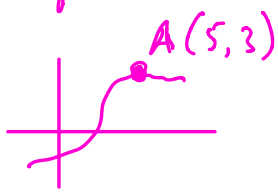
Examples

1)



$y = f(x)$

Where will A go on

$y = f(x-1) + 4$

Translation by $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$A \rightarrow (5+1, 3+4)$

$A' = (6, 7)$

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2) $y = f(x)$

$y = (x-3) - 2$

Translation by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$A(5,3) \longrightarrow A'(5+3,\ 3-2)$

$A'(8,1)$

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3) $y = f(x)$ $\qquad y = f(x+4) - 3$

Translation by $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

$A(5,3) \longrightarrow A'(5-4,\ 3-3)$

$A'(1,0)$

Exercise $\quad$ Find A' when $A(5,3)$

1) $y = f(x)$ $\qquad y = f(x+1) + 1$

Translation by $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\qquad A'(5-1,\ 3+1)$ $\qquad A'(4,4)$

2) $y = f(x-10) + 7$

Translation by $\begin{pmatrix} 10 \\ 7 \end{pmatrix}$ $\qquad A'(5+10,\ 3+7)$ $\qquad A'(15,10)$

3) $y = f(x-5) - 5$

Translation by $\begin{pmatrix} 5 \\ -5 \end{pmatrix}$ $\qquad A'(5+5,\ 3-5)$ $\qquad A'(10,-2)$

4) $y = f(x+6) - 4$

Translation by $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$ $\qquad A'(5-6,\ 3-4)$ $\qquad A'(-1,-1)$
