Binomial and Hypothesis Testing Homework

8 Mark is playing solitaire on his computer. The probability that he wins a game is 0.2, independently of all other games that he plays.

(i) Find the expected number of wins in 12 games. **[2]**

(ii) Find the probability that

(A) he wins exactly 2 out of the next 12 games that he plays, **[3]**

(B) he wins at least 2 out of the next 12 games that he plays. **[3]**

(iii) Mark's friend Ali also plays solitaire. Ali claims that he is better at winning games than Mark. In a random sample of 20 games played by Ali, he wins 7 of them. Write down suitable hypotheses for a test at the 5% level to investigate whether Ali is correct. Give a reason for your choice of alternative hypothesis. Carry out the test. **[9]**

8 The Department of Health 'eat five a day' advice recommends that people should eat at least five portions of fruit and vegetables per day. In a particular school, 20% of pupils eat at least five a day.

(i) 15 children are selected at random.

(A) Find the probability that exactly 3 of them eat at least five a day. **[3]**

(B) Find the probability that at least 3 of them eat at least five a day. **[3]**

(C) Find the expected number who eat at least five a day. **[2]**

A programme is introduced to encourage children to eat more portions of fruit and vegetables per day. At the end of this programme, the diets of a random sample of 15 children are analysed. A hypothesis test is carried out to examine whether the proportion of children in the school who eat at least five a day has increased.

(ii) (A) Write down suitable null and alternative hypotheses for the test.

(B) Give a reason for your choice of the alternative hypothesis. **[4]**

(iii) Find the critical region for the test at the 10% significance level, showing all of your calculations. Hence complete the test, given that 7 of the 15 children eat at least five a day. **[6]**

6 A manufacturer produces tiles. On average 10% of the tiles produced are faulty. Faulty tiles occur randomly and independently.

A random sample of 18 tiles is selected.

(i) (*A*) Find the probability that there are exactly 2 faulty tiles in the sample. **[3]**

(*B*) Find the probability that there are more than 2 faulty tiles in the sample. **[3]**

(*C*) Find the expected number of faulty tiles in the sample. **[2]**

A cheaper way of producing the tiles is introduced. The manufacturer believes that this may increase the proportion of faulty tiles. In order to check this, a random sample of 18 tiles produced using the cheaper process is selected and a hypothesis test is carried out.

(ii) (*A*) Write down suitable null and alternative hypotheses for the test.

(*B*) Explain why the alternative hypothesis has the form that it does. **[4]**

(iii) Find the critical region for the test at the 5% level, showing all of your calculations. **[4]**

(iv) In fact there are 4 faulty tiles in the sample. Complete the test, stating your conclusion clearly. **[2]**

(iii) Mark's friend Ali also plays solitaire. Ali claims that he is better at winning games than Mark. In a random sample of 20 games played by Ali, he wins 7 of them. Write down suitable hypotheses for a test at the 5% level to investigate whether Ali is correct. Give a reason for your choice of alternative hypothesis. Carry out the test. **[9]**

$H_0 : p = 0.2$ p is the prob he wins

$H_1 : p > 0.2$ a randomly chosen game

H_1 chosen as $p > 0.2$ because Ali thinks he has a higher probability

$X \sim B(20, 0.2)$

$P(X \geq 7) = 1 - P(X \leq 6)$

$1 - 0.9133$

$= 0.0867 > 5\%$ Accept H_0 $p = 0.2$

There is not sufficient evidence to support the view Ali has a greater chance of winning

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ii (A) $X \sim B(15, 0.2)$

$H_0: p = 0.2$

$H_1: p > 0.2$

p is prob a randomly chosen child eats 5 a day

H_1 chosen as $p > 0.2$ because looking for increase after encouragement.

$P(X \le 4) = 0.8357$

$P(X \le 5) = 0.9389$

$P(X \geq 5) = 1 - P(X \le 4)$

$= 1 - 0.8357 = 0.1643 > 10\%$

$P(X \geq 6) = 1 - P(X \le 5)$

$= 1 - 0.9389 = 0.0611 < 10\%$

So critical region $= \{6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

7 is in C.R. therefore reject H_0 and accept H_1 $p > 0.2$

There is sufficient evidence to support the view the proportion of children eating 5 a day has increased

A cheaper way of producing the tiles is introduced. The manufacturer believes that this may increase the proportion of faulty tiles. In order to check this, a random sample of 18 tiles produced using the cheaper process is selected and a hypothesis test is carried out.

(ii) (*A*) Write down suitable null and alternative hypotheses for the test.

(*B*) Explain why the alternative hypothesis has the form that it does. **[4]**

(iii) Find the critical region for the test at the 5% level, showing all of your calculations. **[4]**

(iv) In fact there are 4 faulty tiles in the sample. Complete the test, stating your conclusion clearly. **[2]**

$X \sim B(18, 0.1)$

$H_0: p = 0.1$ where p is prob a random tile is faulty

$H_1: p > 0.1$

$H_1: p > 0.1$ since more faulty tiles suspected.

$P(X \leq 3) = 0.9018$

$P(X \leq 4) = 0.9718$

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.9018$

$= 0.0982 > 5\%$

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0.9718$

$= 0.0282 < 5\%$

$C.R. = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

4 not in C.R. Accept H_0 $p = 0.1$

There is not sufficient evidence to suggest the proportion of faulty tiles has increased.