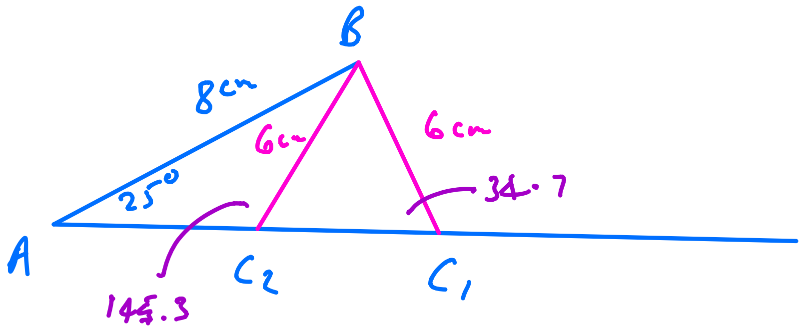
When finding an angle using sine rule, sometimes there are two possibilities




Find angle C in the △

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{c \sin A}{a} = \sin C$$

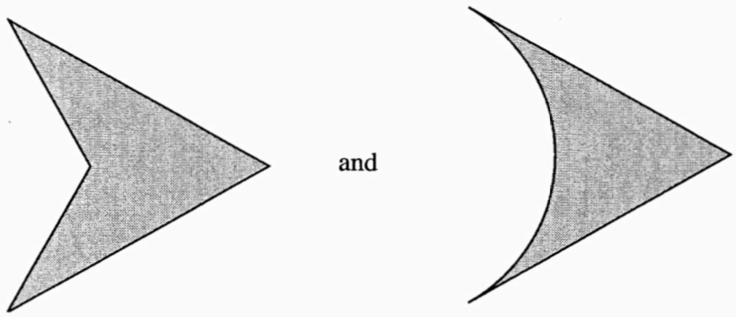
$$\sin C = \frac{8 \sin 25}{6} = 0.56349$$

$$C = \sin^{-1}(0.56349) = 34.3^{\circ}$$

However, $180 - 34.3^{\circ} = 145.7^{\circ}$ is also a possible answer

Shown above

**10** *Arrowline Enterprises* is considering two possible logos:



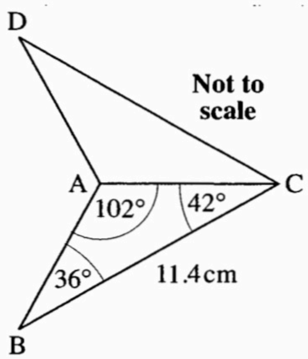




**Fig. 10.1**

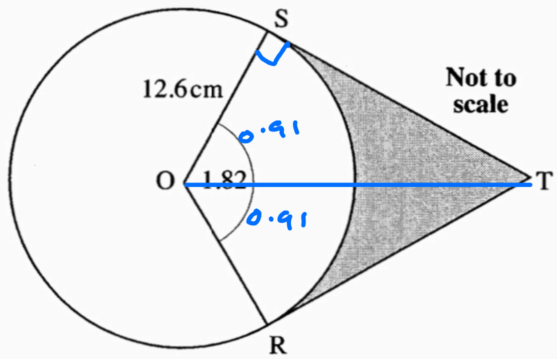



**Fig. 10.2**

**(i)** Fig. 10.1 shows the first logo ABCD. It is symmetrical about AC.

Find the length of AB and hence find the area of this logo. [4]

**(ii)** Fig. 10.2 shows a circle with centre O and radius 12.6 cm. ST and RT are tangents to the circle and angle SOR is 1.82 radians. The shaded region shows the second logo.

Show that ST = 16.2 cm to 3 significant figures.

Find the area and perimeter of this logo. [8]

i) Sine Rule $\frac{AB}{\sin 42} = \frac{11.4}{\sin 102}$

$AB = \frac{11.4}{\sin 102^\circ} \times \sin 42 = 7.7985$

$\underline{AB = 7.80 \text{ cm}}$

Area of $\triangle ABC = \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 11.4 \times 7.7985 \sin 36^\circ$

$= 26.128 \text{ cm}^2$

Area of logo $= 2 \times 26.128$

$= 52.3 \text{ cm}^2$ to 3 s.f.

ii) $\tan 0.91 = \frac{ST}{12.6}$

$ST = \tan 0.91 \times 12.6 = 16.208$

$= 16.2 \text{ cm}$ to 3 s.f

Perimeter $=$ arc length $+ 16.2 + 16.2$

$= r\theta + 32.4$

$= 12.6 \times 1.82 + 32.4$

$= 55.3 \text{ cm}$

Find area

Area of $\triangle OST = \frac{1}{2} \times 12.6 \times 16.2$

$= 102.06$

Area of quad $OSTR = 2 \times 102.06$

$= 204.12$

Area of sector $OSR = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 12.6^2 \times 1.82$

$= 144.47 \text{ cm}^2$

Area of logo $= 204.12 - 144.47$

$= 59.65 = 59.7 \text{ cm}^2$

**10** **(i)**

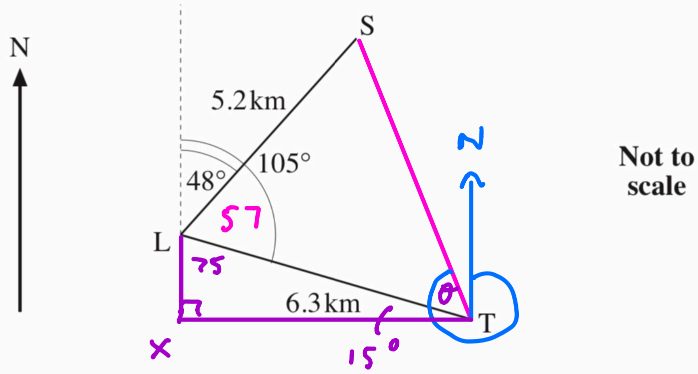



**Fig. 10.1**

At a certain time, ship S is 5.2 km from lighthouse L on a bearing of 048°. At the same time, ship T is 6.3 km from L on a bearing of 105°, as shown in Fig. 10.1.

For these positions, calculate

(*A*) the distance between ships S and T, [3]

(*B*) the bearing of S from T. [3]

$$ST^2 = 5.2^2 + 6.3^2 - 2 \times 5.2 \times 6.3 \cos 57$$ cosine rule

$$ST = 5.57 \text{ km}$$

$$\angle XLT = 180 - 105 = 75^\circ$$
$$\angle XTL = 90 - 75 = 15^\circ$$

In $\triangle LST$

$$\frac{5.57}{\sin 57^\circ} = \frac{5.2}{\sin\theta}$$

$$\frac{\sin 57^\circ}{5.57} = \frac{\sin\theta}{5.2}$$

$$\frac{\sin 57^\circ}{5.57} \times 5.2 = \sin\theta$$

$$\sin\theta = 0.783$$

$$\theta = \sin^{-1}(0.783) = 51.5^\circ$$

$$\text{Bearing} = 270^\circ + 15^\circ + 51.5^\circ = 336.5^\circ$$

**(ii)**

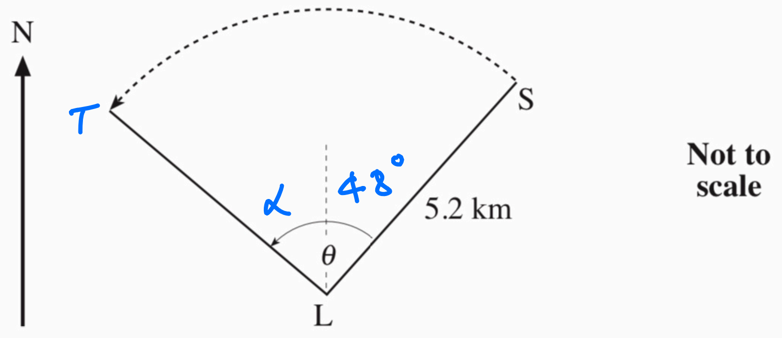



**Fig. 10.2**

Ship S then travels at 24 km h$^{-1}$ anticlockwise along the arc of a circle, keeping 5.2 km from the lighthouse L, as shown in Fig. 10.2.

Find, in radians, the angle $\theta$ that the line LS has turned through in 26 minutes.

Hence find, in degrees, the bearing of ship S from the lighthouse at this time. [5]

$$\text{Arc } ST = 24 \times \frac{26}{60} = 10.4 \text{ km}$$

$$\text{Arc} = r\theta$$

$$10.4 = 5.2\,\theta$$

$$\underline{\theta = 2 \text{ radians}}$$

$$2 \text{ radians} = 2 \times \frac{180}{\pi} \text{ degrees} = 114.6^\circ$$

$$\alpha = \theta - 48$$

$$\alpha = 114.6 - 48$$

$$\alpha = 66.6^\circ$$

Bearing of Ship from Lighthouse

$$= 360 - \alpha$$

$$= 360 - 66.6 = \underline{293.4^\circ}$$