Non-Linear Simultaneous Equations Exam Questions

Questions

Q1.

Solve the simultaneous equations

$$x+y=2$$
$$4y^2-x^2=11$$

(7)

(Total 7 marks)

Q2.

Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(7)

(Total 7 marks)

Q3.

Given the simultaneous equations

$$2x + y = 1$$

$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

(Total 8 marks)

Q4.

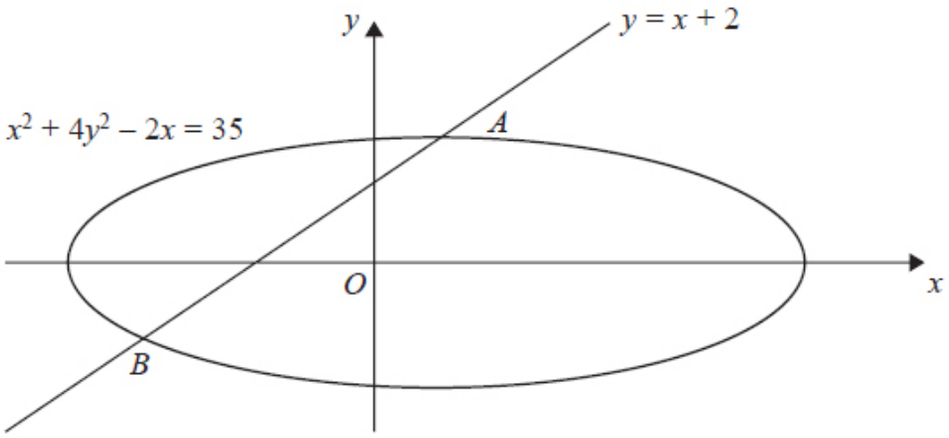

Figure 2

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

(a) Find the coordinates of A and the coordinates of B.

(6)

(b) Find the distance AB in the form $r\sqrt{2}$ where r is a rational number.

(3)

(Total 9 marks)

Q5.

Solve the simultaneous equations

$$y - 2x - 4 = 0$$

$$4x^2 + y^2 + 20x = 0$$

(7)

(Total for question = 7 marks)

Q6.

(a) By eliminating y from the equations

$$y = x - 4$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

(Total 7 marks)

Q7.

Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

(Total for question = 6 marks)

Q8.

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$,

where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2.

(3)

(Total 7 marks)

Q9.

The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

(a) Sketch the graphs of C and l, indicating clearly the coordinates of any intersections with the axes.

(3)

(b) Find the coordinates of the points of intersection of C and l.

(6)

(Total 9 marks)

Q10.

The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

(a) Show that $4p^2 - 20p + 9 < 0$

(4)

(b) Hence find the set of possible values of p.

(4)

(Total for question = 8 marks)

Q11.

(a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 2$

(3)

(c) Hence find the range of possible values for c.

(4)

(Total for question = 11 marks)

Q12.

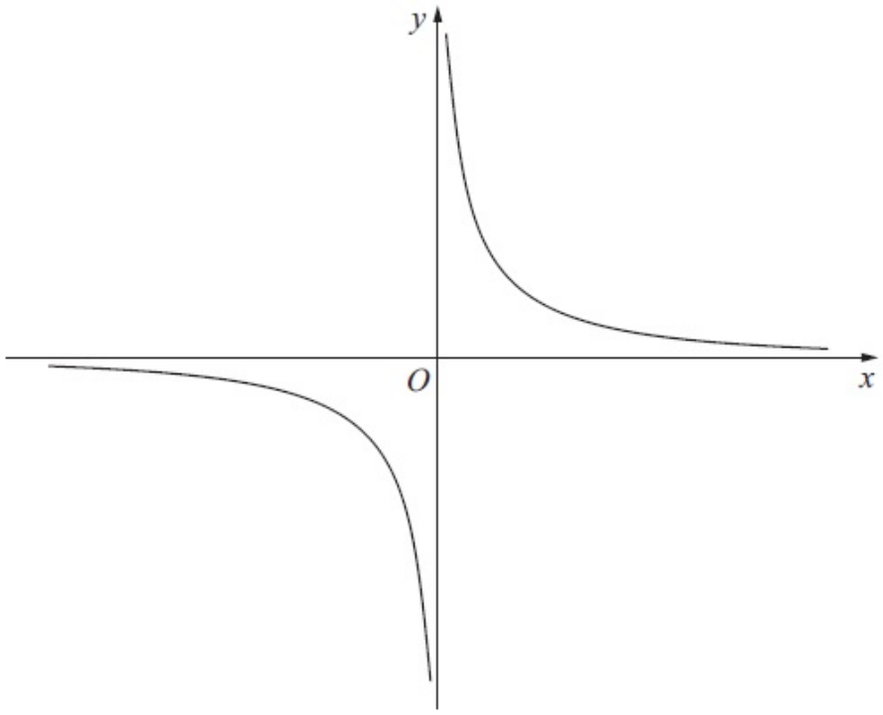

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}$, $x \neq 0$

The curve C has equation $y = \frac{2}{x} - 5$, $x \neq 0$, and the line l has equation $y = 4x + 2$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

(5)

(b) Write down the equations of the asymptotes of the curve C.

(2)

(c) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$

(5)

(Total 12 marks)

Q2.

Solve the simultaneous equations

$$y - 3x + 2 = 0 \quad (1)$$

$$y^2 - x - 6x^2 = 0 \quad (2)$$

(7)

(Total 7 marks)

From (1) $y = 3x - 2$

Sub for y in (2)

$$(3x-2)^2 - x - 6x^2 = 0$$

$$9x^2 + 4 - 12x - x - 6x^2 = 0$$

$$3x^2 - 13x + 4 = 0$$

3×4
$+12$
$-1 \quad -12$

$$3x^2 - x - 12x + 4 = 0$$

$$x(3x-1) - 4(3x-1) = 0$$

$$(x-4)(3x-1) = 0$$

Either $x - 4 = 0$ or $3x - 1 = 0$

$x = 4$

$3x = 1$

$x = \frac{1}{3}$

when $x = 4$

$y = 3(4) - 2$

$y = 10$

when $x = \frac{1}{3}$

$y = 3(\frac{1}{3}) - 2$

$y = -1$

$\begin{cases} x = 4 \\ y = 10 \end{cases}$ $\begin{cases} x = \frac{1}{3} \\ y = -1 \end{cases}$

Q4.

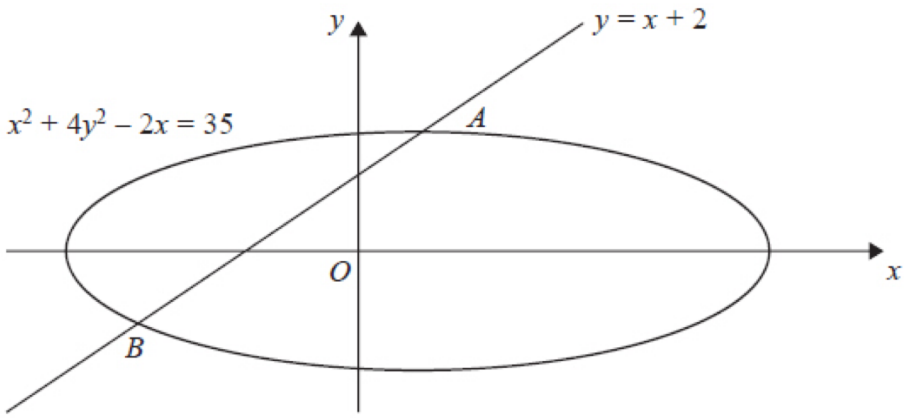

Figure 2

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

(a) Find the coordinates of A and the coordinates of B.

(6)

(b) Find the distance AB in the form $r\sqrt{2}$ where r is a rational number.

(3)

a)

$y = x + 2$ (1)

$x^2 + 4y^2 - 2x = 35$ (2)

Sub for y in (2)

$x^2 + 4(x+2)^2 - 2x = 35$

$x^2 + 4[x^2 + 4 + 4x] - 2x - 35 = 0$

$x^2 + 4x^2 + 16 + 16x - 2x - 35 = 0$

$5x^2 + 14x - 19 = 0$

$x = 1$ or $x = -\frac{19}{5}$

$y = 1 + 2$ $\quad$ $y = -\frac{19}{5} + 2$

$y = 3$ $\quad$ $y = -\frac{9}{5}$

$\begin{cases} x = 1 \\ y = 3 \end{cases}$ $\quad$ $x = -\frac{19}{5}$, $y = -\frac{9}{5}$

$A(1,3)$ $B\left(-\frac{19}{5}, -\frac{9}{5}\right)$

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b) $|AB| = \sqrt{\left(1--\frac{19}{5}\right)^2 + \left(3--\frac{9}{5}\right)^2}$

$= \sqrt{\left(\frac{24}{5}\right)^2 + \left(\frac{24}{5}\right)^2}$

$= \sqrt{2\left(\frac{24}{5}\right)^2}$

$\frac{24}{5}\sqrt{2}$

Q6.

(a) By eliminating y from the equations

$$y = x - 4 \quad (1)$$

$$2x^2 - xy = 8, \quad (2)$$

show that

$$x^2 + 4x - 8 = 0$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

a) Sub for y in (2)

$2x^2 - x(x-4) = 8$

$2x^2 - x^2 + 4x = 8$

$$x^2 + 4x - 8 = 0$$

b)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times -8}}{2}$$

$$x = \frac{-4 \pm \sqrt{48}}{2}$$

$$x = \frac{-4 \pm \sqrt{16 \times 3}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{3}}{2}$$

$$x = -2 \pm 2\sqrt{3}$$

Q1.

Solve the simultaneous equations

$$x + y = 2 \quad (1)$$
$$4y^2 - x^2 = 11 \quad (2)$$

(7)

From (1) $y = 2 - x$

Sub for y in (2)

$$4(2-x)^2 - x^2 = 11$$
$$4(4 + x^2 - 4x) - x^2 - 11 = 0$$
$$16 + 4x^2 - 16x - x^2 - 11 = 0$$

(Total 7 marks)

$$3x^2 - 16x + 5 = 0$$

$$(3x - 1)(x - 5) = 0$$

Either $3x - 1 = 0$ or $x - 5 = 0$

$3x = 1$

$x = \frac{1}{3}$ $\qquad$ $x = 5$

$y = 2 - \frac{1}{3}$ $\qquad$ $y = 2 - 5$

$y = \frac{5}{3}$ $\qquad$ $y = -3$

$\begin{cases} x = \frac{1}{3} \\ y = \frac{5}{3} \end{cases}$ $\qquad$ $\begin{cases} x = 5 \\ y = -3 \end{cases}$

Q3.

Given the simultaneous equations

$$2x + y = 1 \quad (1)$$

$$x^2 - 4ky + 5k = 0 \quad (2)$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

a) From ① $y = 1 - 2x$

Sub for y in ②

$$x^2 - 4k(1-2x) + 5k = 0$$

$$x^2 - 4k + 8kx + 5k = 0$$

$$x^2 + 8kx + k = 0$$

b) Equal roots $\Rightarrow b^2 - 4ac = 0$

$$\Rightarrow (8k)^2 - 4 \times 1 \times k = 0$$

$$64k^2 - 4k = 0$$

$$4k(16k - 1) = 0$$

$k \neq 0$ $\therefore 16k - 1 = 0$

$$16k = 1$$

$$k = \frac{1}{16}$$

c)

$$x^2 + \frac{8}{16}x + \frac{1}{16} = 0$$

$$16x^2 + 8x + 1 = 0$$

$$(4x + 1)^2 = 0$$

$$4x + 1 = 0$$

$$4x = -1$$

$$x = -\frac{1}{4}$$

$$y = 1 + \frac{2}{4} \qquad y = \frac{3}{2}$$

$$\begin{cases} x = -\frac{1}{4} \\ y = \frac{3}{2} \end{cases}$$

Q5.

Solve the simultaneous equations

$$y - 2x - 4 = 0 \quad (1)$$
$$4x^2 + y^2 + 20x = 0 \quad (2)$$

(7)

From (1) $y = 2x + 4$

(Total for question = 7 marks)

Sub for y in (2)

$$4x^2 + (2x+4)^2 + 20x = 0$$
$$4x^2 + 4x^2 + 16 + 16x + 20x = 0$$
$$8x^2 + 36x + 16 = 0$$
$$2x^2 + 9x + 4 = 0$$
$$(2x+1)(x+4) = 0$$

$\Rightarrow$ $2x + 1 = 0$ or $x + 4 = 0$

$2x = -1$

$x = -\frac{1}{2}$

$x = -4$

$y = 2(-\frac{1}{2}) + 4$

$y = 3$

$y = 2(-4) + 4$

$y = -4$

$\begin{cases} x = -\frac{1}{2} \\ y = 3 \end{cases}$

$\begin{cases} x = -4 \\ y = -4 \end{cases}$