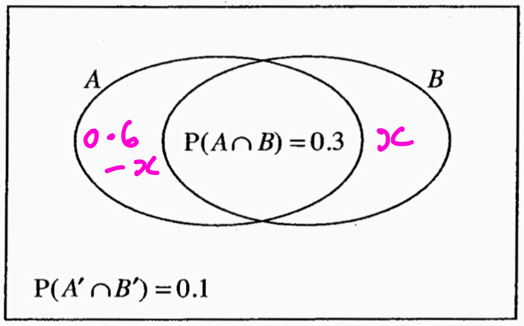
3 The Venn diagram illustrates the occurrence of two events A and B.



You are given that $\mathrm{P}(A \cap B) = 0.3$ and that the probability that neither A nor B occurs is 0.1. You are also given that $\mathrm{P}(A) = 2\mathrm{P}(B)$.

Find $\mathrm{P}(B)$. [3]

$P(A) = 2P(B)$

$0.6 - x + 0.3 = 2(0.3 + x)$

$0.9 - x = 0.6 + 2x$

$0.9 - 0.6 = 3x$

$0.3 = 3x$

$x = 0.1$

$\underline{P(B) = 0.4}$

6 An amateur weather forecaster describes each day as either sunny, cloudy or wet. He keeps a record each day of his forecast and of the actual weather. His results for one particular year are given in the table.

		Weather Forecast			Total
		Sunny	Cloudy	Wet	
Actual Weather	Sunny	55	12	7	74
	Cloudy	17	128	29	174
	Wet	3	33	81	117
Total		75	173	117	365

A day is selected at random from that year.

(i) Show that the probability that the forecast is correct is $\frac{264}{365}$. [2]

Find the probability that

$P(\text{correct}) = \frac{55 + 128 + 81}{365} = \frac{264}{365}$

(ii) the forecast is correct, given that the forecast is sunny, [2]

$\frac{55}{75}$

(iii) the forecast is correct, given that the weather is wet, [2]

$\frac{81}{117}$

(iv) the weather is cloudy, given that the forecast is correct. [2]

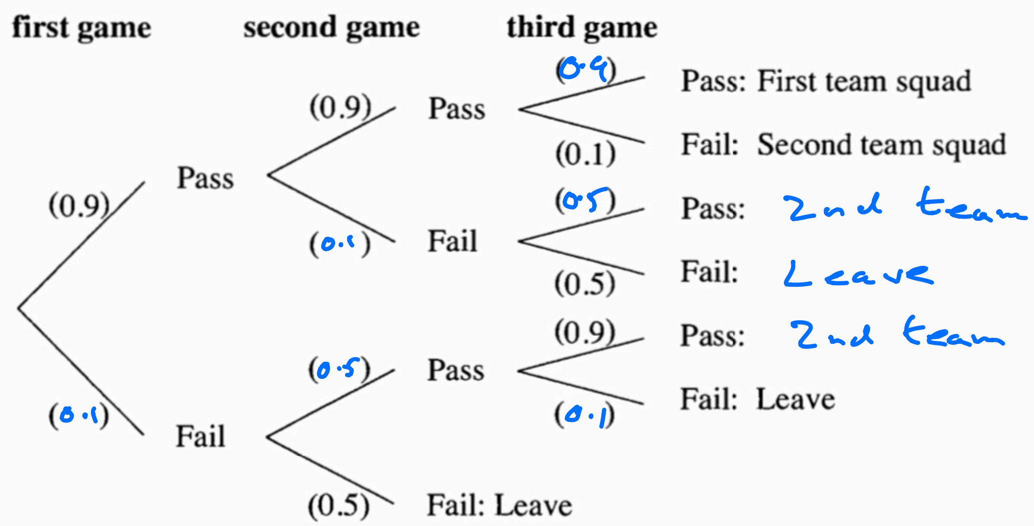
$\frac{128}{264}$

6 Answer part (i) of this question on the insert provided.

Mancaster Hockey Club invite prospective new players to take part in a series of three trial games. At the end of each game the performance of each player is assessed as pass or fail. Players who achieve a pass in all three games are invited to join the first team squad. Players who achieve a pass in two games are invited to join the second team squad. Players who fail in two games are asked to leave. This may happen after two games.

- The probability of passing the first game is 0.9
- Players who pass any game have probability 0.9 of passing the next game
- Players who fail any game have probability 0.5 of failing the next game

(i) On the insert, complete the tree diagram which illustrates the information above. [2]



(ii) Find the probability that a randomly selected player

(*A*) is invited to join the first team squad, $0.9^3 = 0.729$ [2]

(*B*) is invited to join the second team squad. $0.9^2 \times 0.1 + 0.9 \times 0.1 \times 0.5 + 0.1 \times 0.5 \times 0.9 = 0.171$ [3]

(iii) Hence write down the probability that a randomly selected player is asked to leave. [1]

0.1

(iv) Find the probability that a randomly selected player is asked to leave after two games, given that the player is asked to leave. [2]

Angela, Bryony and Shareen attend the trials at the same time. Assuming their performances are independent, find the probability that

(v) at least one of the three is asked to leave, [3]

(vi) they pass a total of 7 games between them. [5]

iv) P(Leave after 2 games \ Leave)

$$\frac{P(\text{Leave after} \cap \text{Leave})}{P(\text{Leave})} = \frac{0.1 \times 0.5}{0.1} = 0.5$$

v) $X \sim B(3, 0.1)$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.9^3$

$= 0.271$

---

vi)

A	B	S
3	3	1
3	2	2

$P(3) \times P(3) \times P(1)$

$.9^3 \times .9^3 \times (.1 \times .9 \times .5 + .1 \times .5 \times .1)$

0.02657205

$P(3) \times P(2) \times P(2)$

$.9^3 \times .171 \times .171$

$= 0.021316689$

3 ways for 331   3 ways for 322

$P(\text{Win 7 games}) = 3 \times 0.02657205 + 3 \times 0.021316689$

$= 0.1437$

Homework

5 A school athletics team has 10 members. The table shows which competitions each of the members can take part in.

		Competiton				
		100 m	200 m	110 m hurdles	400 m	Long jump
Athlete	Abel	✓	✓			✓
	Bernoulli		✓		✓	
	Cauchy	✓		✓		✓
	Descartes	✓	✓			
	Einstein		✓		✓	
	Fermat	✓		✓		
	Galois				✓	✓
	Hardy	✓	✓			✓
	Iwasawa		✓		✓	
	Jacobi			✓		

An athlete is selected at random. Events A, B, C, D are defined as follows.

A: the athlete can take part in exactly 2 competitions.
B: the athlete can take part in the 200 m.
C: the athlete can take part in the 110 m hurdles.
D: the athlete can take part in the long jump.

(i) Write down the value of $\mathrm{P}(A \cap B)$. [1]

(ii) Write down the value of $\mathrm{P}(C \cup D)$. [1]

(iii) Which two of the four events A, B, C, D are mutually exclusive? [1]

(iv) Show that events B and D are not independent. [2]

5 Each day the probability that Ashwin wears a tie is 0.2. The probability that he wears a jacket is 0.4. If he wears a jacket, the probability that he wears a tie is 0.3.

(i) Find the probability that, on a randomly selected day, Ashwin wears a jacket and a tie. [2]

(ii) Draw a Venn diagram, using one circle for the event 'wears a jacket' and one circle for the event 'wears a tie'. Your diagram should include the probability for each region. [3]

(iii) Using your Venn diagram, or otherwise, find the probability that, on a randomly selected day, Ashwin

(A) wears either a jacket or a tie (or both),

(B) wears no tie or no jacket (or wears neither). [3]

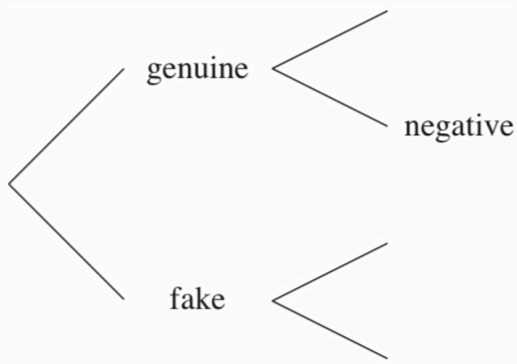
Section B (36 marks)

6 It has been estimated that 90% of paintings offered for sale at a particular auction house are genuine, and that the other 10% are fakes. The auction house has a test to determine whether or not a given painting is genuine. If this test gives a positive result, it suggests that the painting is genuine. A negative result suggests that the painting is a fake.

If a painting is genuine, the probability that the test result is positive is 0.95.

If a painting is a fake, the probability that the test result is positive is 0.2.

(i) Copy and complete the probability tree diagram below, to illustrate the information above. [2]



Calculate the probabilities of the following events.

(ii) The test gives a positive result. [2]

(iii) The test gives a correct result. [2]

(iv) The painting is genuine, given a positive result. [3]

(v) The painting is a fake, given a negative result. [3]

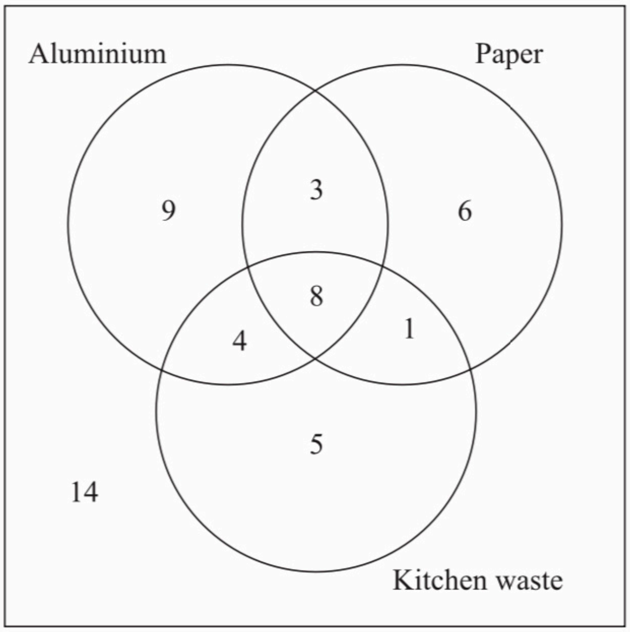
A second test is more accurate, but very expensive. The auction house has a policy of only using this second test on those paintings with a negative result on the original test.

(vi) Using your answers to parts **(iv)** and **(v)**, explain why the auction house has this policy. [2]

The probability that the second test gives a correct result is 0.96 whether the painting is genuine or a fake.

(vii) Three paintings are independently offered for sale at the auction house. Calculate the probability that all three paintings are genuine, are judged to be fakes in the first test, but are judged to be genuine in the second test. [4]

4 A local council has introduced a recycling scheme for aluminium, paper and kitchen waste. 50 residents are asked which of these materials they recycle. The numbers of people who recycle each type of material are shown in the Venn diagram.



One of the residents is selected at random.

(i) Find the probability that this resident recycles

(A) at least one of the materials, [1]

(B) exactly one of the materials. [2]

(ii) Given that the resident recycles aluminium, find the probability that this resident does not recycle paper. [2]

Two residents are selected at random.

(iii) Find the probability that exactly one of them recycles kitchen waste. [3]