# Iteration

**3** **a** Use the 'Babylonian' iterative formula

$$x_{n+1} = \frac{x_n}{2} + \frac{2}{2x_n}$$

to find a fraction approximation to $\sqrt{2}$. Use three iterations starting with the estimate $x_1 = 1$.

**b** What is the result of squaring your answer to **a**?

**Did you know...**

Some historians believe that 4000 years ago Babylonian mathematicians used iterative formula to find the square roots of numbers.

$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}$$

$$x_1 = 1$$

$$x_2 = \frac{1}{2} + \frac{1}{1} = 1.5$$

$$x_3 = \frac{1.5}{2} + \frac{1}{1.5} = 1.417$$

$$x_4 = \frac{1.417}{2} + \frac{1}{1.417} = 1.4142$$

$$\sqrt{2} \approx 1.4142$$

$$(1.4142)^2 = 1.99996$$

so it is a very good approximation to $\sqrt{2}$

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**4** **a** Use the 'Babylonian' iterative formula

$$x_{n+1} = \frac{x_n}{2} + \frac{3}{2x_n}$$

to find a fraction approximation to $\sqrt{3}$. Use two iterations starting with the estimate $x_1 = 2$.

**b** What happens if you use the much simpler formula $x_{n+1} = \frac{3}{x_n}$?

$$x_{n+1} = \frac{x_n}{2} + \frac{3}{2x_n}$$

$$x_1 = 2$$

$$x_2 = \frac{2}{2} + \frac{3}{(2 \times 2)} = 1.75$$

$$x_3 = \frac{1.75}{2} + \frac{3}{(2 \times 1.75)} = 1.732$$

$$x_4 = \frac{1.732}{2} + \frac{3}{(2 \times 1.732)} = 1.732$$

$$\sqrt{3} \approx 1.732$$

b) $x_{n+1} = \frac{3}{x_n}$

$x_1 = 2$

$x_2 = \frac{3}{2} = 1.5$

$x_3 = \frac{3}{1.5} = 2$

Formula would give results that do not converge to $\sqrt{3}$ but oscillate between 1.5 and 2

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**5** In 2010, a survey of the birds on an island counted approximately 200 kittiwakes.

A conservationist used the logistic equation

$$P_{n+1} = P_n(1.4 - 0.001P_n)$$

to predict the expected population, $P_n$, $n$ years later.

**a** What did the equation predict for the size of the colony of kittiwakes each year from 2011 to 2015?

**b** Describe in your own words how the size of the colony was expected to change.

**c** Assuming that the size of the colony will stabilise at a roughly constant value, find this equilibrium size.

$P_{n+1} = P_n(1.4 - 0.001P_n)$

2010 200

2011 $200(1.4 - 0.001 \times 200)$

$= 240$

2012 $240(1.4 - 0.001 \times 240)$

$= 278.4 = 279$

2013 $279(1.4 - 0.001 \times 279)$

$= 312.759 = 313$

2014 $313(1.4 - 0.001 \times 313)$

$= 340.231 = 340$

2015 $340(1.4 - 0.001 \times 340)$

$= 360.4 = 360$

Size of colony in 2015 predicted to be 360

b) Population is increasing but rate of increase gradually slows down.

c) Population will stabilise when $P_n$ is being multiplied by 1

$$\text{so } (1.4 - 0.001P_n) = 1$$

$$1.4 = 1 + 0.001P_n$$

$$0.4 = 0.001P_n$$

$$\frac{0.4}{0.001} = P_n$$

$$P_n = 400$$

Stabilses at around 400

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