

7. A curve C has equation

$$y = 3 \sin 2x + 4 \cos 2x, \quad -\pi \leq x \leq \pi.$$

The point $A(0, 4)$ lies on C .

(a) Find an equation of the normal to the curve C at A .

(5)

(b) Express y in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x -axis.
Give your answers to 2 decimal places.

(4)

a)
$$\frac{dy}{dx} = 6 \cos 2x - 8 \sin 2x$$

At $A(0, 4)$
$$\frac{dy}{dx} = 6 \cos 0 - 8 \sin 0 = 6$$

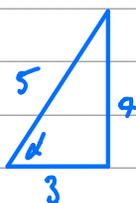
Gradient of normal = $-\frac{1}{6}$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{6}(x - 0)$$

$$y = -\frac{1}{6}x + 4$$

b)
$$y = 3 \sin 2x + 4 \cos 2x$$



$$y = 5 \left(\frac{3}{5} \sin 2x + \frac{4}{5} \cos 2x \right)$$

$$y = 5 \sin(2x + \alpha)$$

where $\alpha = \tan^{-1} \frac{4}{3}$

$$y = 5 \sin(2x + 0.927)$$



Question 7 continued

c) Domain $-\pi \leq x \leq \pi$

Intersects x -axis when

$$5 \sin(2x + 0.927) = 0$$

$$\Rightarrow 2x + 0.927 = \cancel{-2\pi}, -\pi, 0, \pi, 2\pi$$

$$x = \frac{-\pi - 0.927}{2} = -2.03$$

$$x = \frac{0 - 0.927}{2} = -0.46$$

$$x = \frac{\pi - 0.927}{2} = 1.11$$

$$x = \frac{2\pi - 0.927}{2} = 2.68$$

Points are:

$$(-2.03, 0), (-0.46, 0), (1.11, 0), (2.68, 0)$$



2.

$$f(x) = 5 \cos x + 12 \sin x$$

Given that $f(x) = R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places. (4)

(b) Hence solve the equation

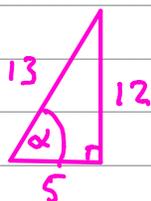
$$5 \cos x + 12 \sin x = 6$$

for $0 \leq x < 2\pi$. (5)

(c) (i) Write down the maximum value of $5 \cos x + 12 \sin x$. (1)

(ii) Find the smallest positive value of x for which this maximum value occurs. (2)

a)
$$f(x) = 13 \left(\frac{5}{13} \cos x + \frac{12}{13} \sin x \right)$$



$$f(x) = 13 \cos(x - 1.176)$$

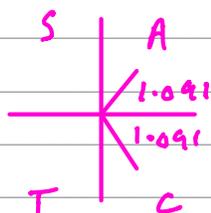
$$\alpha = \tan^{-1} \frac{12}{5}$$

$$\alpha = 1.176$$

b)
$$13 \cos(x - 1.176) = 6$$

$$\cos(x - 1.176) = \frac{6}{13}$$

$$x - 1.176 = \cos^{-1}\left(\frac{6}{13}\right) = 1.091, 5.192$$



$$x = 1.091 + 1.176, 5.192 + 1.176$$

$$x = 2.267, 6.368 \quad (> 2\pi)$$

out of range
so subtract 2π



Question 2 continued

$$x = 2.267, \quad 0.085$$

c) i) Max value = 13

ii) Occurs when $13 \cos(x - 1.176) = 1$

First positive value occurs when

$$x - 1.176 = 0$$

$$x = 1.176$$



6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta. \tag{4}$$

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π . (5)

(b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2}). \tag{4}$$

a) i) $\sin 3\theta = \sin(2\theta + \theta)$

$$= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$$

$$= 2 \sin \theta \cos \theta \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta$$

$$= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$$

$$= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$$

$$= 3 \sin \theta - 4 \sin^3 \theta$$

ii) $8 \sin^3 \theta - 6 \sin \theta + 1 = 0$

$$-2(-4 \sin^3 \theta + 3 \sin \theta) + 1 = 0$$

$$-2 \sin 3\theta + 1 = 0$$

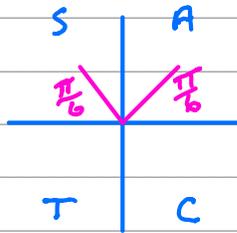
$$1 = 2 \sin 3\theta$$

$$\frac{1}{2} = \sin 3\theta$$

$$3\theta = \sin^{-1} \frac{1}{2}$$



Question 6 continued



$$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$$

b)

$$\sin(\theta - \alpha) = \sin\theta \cos\alpha - \cos\theta \sin\alpha$$

$$\sin 15^\circ = \sin(60 - 45) = \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$$

$$\sin 15^\circ = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$\sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$\sin 15^\circ = \frac{\sqrt{2}(\sqrt{3} - 1)}{\sqrt{2} \times 2\sqrt{2}}$$

$$\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$$



6. (a) Use the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A \tag{2}$$

The curves C_1 and C_2 have equations

$$C_1: y = 3\sin 2x$$

$$C_2: y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x -coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2 \tag{3}$$

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leq x < 180^\circ$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

a) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\cos 2A = \cos(A+A) = \cos A \cos A - \sin A \sin A$$

$$= \cos^2 A - \sin^2 A$$

$$= 1 - \sin^2 A - \sin^2 A$$

$$= 1 - 2\sin^2 A$$

b) At intersection

$$3\sin 2x = 4\sin^2 x - 2\cos 2x$$

$$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$$

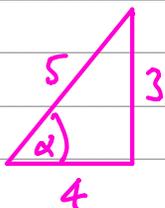


Question 6 continued

$$3 \sin 2x = 2 - 4 \cos 2x$$

$$4 \cos 2x + 3 \sin 2x = 2$$

c)



$$\alpha = \tan^{-1} \frac{3}{4}$$

$$\alpha = 36.87^\circ$$

$$4 \cos 2x + 3 \sin 2x$$

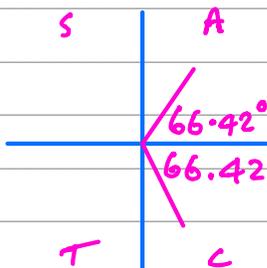
$$= 5 \left(\frac{4}{5} \cos 2x + \frac{3}{5} \sin 2x \right)$$

$$= 5 \cos(2x - 36.87^\circ)$$

d)

$$5 \cos(2x - 36.87^\circ) = 2$$

$$\cos(2x - 36.87^\circ) = \frac{2}{5}$$



$$\cos^{-1}\left(\frac{2}{5}\right) = 66.42^\circ$$

$$2x - 36.87^\circ = 66.42^\circ, 293.58^\circ$$

$$2x = 103.29^\circ, 330.45^\circ$$

$$x = 51.6^\circ, 165.2^\circ$$



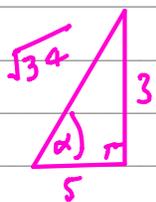
3. (a) Express $5 \cos x - 3 \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. (4)

(b) Hence, or otherwise, solve the equation

$$5 \cos x - 3 \sin x = 4$$

for $0 \leq x < 2\pi$, giving your answers to 2 decimal places. (5)

a)
$$5 \cos x - 3 \sin x = \sqrt{34} \left(\frac{5}{\sqrt{34}} \cos x - \frac{3}{\sqrt{34}} \sin x \right)$$



$$\alpha = \tan^{-1}\left(\frac{3}{5}\right)$$

$$= \sqrt{34} \cos(x + \alpha)$$

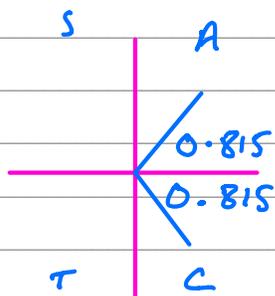
$$= \sqrt{34} \cos(x + 0.540)$$

$$\alpha = 0.540$$

b)
$$\sqrt{34} \cos(x + 0.540) = 4$$

$$\cos(x + 0.540) = \frac{4}{\sqrt{34}}$$

$$x + 0.540 = \cos^{-1}\left(\frac{4}{\sqrt{34}}\right)$$



$$x + 0.540 = 0.815, 5.468$$

$$x = 0.275, 4.928$$

$$\cos^{-1}\left(\frac{4}{\sqrt{34}}\right) = 0.815$$

$$x = 0.28, 4.93$$



7. (a) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(b) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ , for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

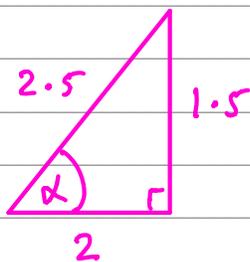
(c) Calculate the maximum value of H predicted by this model and the value of t , to 2 decimal places, when this maximum occurs.

(3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

a) $2 \sin \theta - 1.5 \cos \theta = 2.5 \left(\frac{2}{2.5} \sin \theta - \frac{1.5}{2.5} \cos \theta \right)$



$= 2.5 \sin(\theta - 0.6435)$

b) i) Max Value = 2.5

$\alpha = \tan^{-1} \frac{1.5}{2}$

ii) Occurs when

$\alpha = 0.6435$

$\theta - 0.6435 = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} + 0.6435$

$\theta = 2.2143$



Question 7 continued

$$c) \quad H = 6 + 2.5 \sin\left(\frac{4\pi t}{25} - 0.6435\right)$$

Maximum $H = 8.5$

Occurs when $\frac{4\pi t}{25} - 0.6435 = \frac{\pi}{2}$

$$\frac{4\pi t}{25} = \frac{\pi}{2} + 0.6435$$

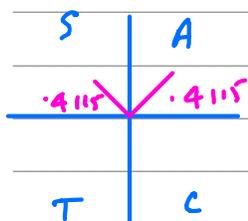
$$t = \frac{25\left(\frac{\pi}{2} + 0.6435\right)}{4\pi}$$

$t = 4.41$

$$d) \quad 6 + 2.5 \sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$$

$$\sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5}$$

$$\frac{4\pi t}{25} - 0.6435 = \sin^{-1}\left(\frac{1}{2.5}\right)$$



$$\frac{4\pi t}{25} - 0.6435 = 0.4115, 2.7301$$

$$\frac{4\pi t}{25} = 1.055, 3.3736$$

$$\sin^{-1}\frac{1}{2.5} = 0.4115$$

$$t = \frac{25 \times 1.055}{4\pi}, \frac{25 \times 3.3736}{4\pi}$$



$$t = 2.0989 \quad 6.7116$$

$$t = 2.06 \text{ pm} \quad 6.43 \text{ pm}$$

