Binomial Distribution

1. Suppose the probability of success in a trial is constant
2. Suppose trials are independent.

This situation can be modelled by the Binomial Distribution

Ex Find the probability of obtaining 2 '5's in 5 rolls of dice

Define 5 as success S

Define all other numbers as failure F

Then $P(SSFFF) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

But $P(SFSFF) = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

How many ways can 2 be chosen from 5 when order does not matter

$${}^5C_2 \text{ or } {}_5C_2 \text{ or } \binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5.4.3.2.1}{2.1\ 3.2.1} = 10$$

$$P(\text{Exactly 2 fives}) = 10 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^3 = \frac{625}{3888} = 0.1608$$

More Generally

If the probability of success in an individual trial is p and the number of trials is n, then the probability of exactly r successes is given by

$$X \sim B(n, p)$$

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

where $q = 1-p$

Examples

1) Let $X \sim B(\overset{n}{10}, \overset{p}{\tfrac{1}{2}})$ find $P(X=5)$

$$P(X=5) = {}^{10}C_{5} \times \left(\tfrac{1}{2}\right)^{5} \times \left(\tfrac{1}{2}\right)^{5}$$

$$= \frac{63}{256} = 0.2461$$

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2) Let $X \sim B(20, 0.7)$ find $P(X=14)$

$$P(X=14) = {}^{20}C_{14} \times 0.7^{14} \times 0.3^{6} = 0.1916$$

Let $X \sim B(\overset{n}{10}, \overset{p}{0.4})$ Find $P(3 \leq X \leq 5)$

$P(X \le 5) - P(X \le 2)$

$0.8338 - 0.1673 = 0.6665$

Alternatively $P(X=3) + P(X=4) + P(X=5)$

$= 0.2149 + 0.2508 + 0.2006$

$= 0.6663$

8 At a doctor's surgery, records show that 20% of patients who make an appointment fail to turn up. During afternoon surgery the doctor has time to see 16 patients.

There are 16 appointments to see the doctor one afternoon.

(i) Find the probability that all 16 patients turn up. [2]

(ii) Find the probability that more than 3 patients do not turn up. [3]

i) Let X represent number of patients who fail to turn up

$X \sim B(\overset{n}{16}, \overset{p}{0.2})$

$P(X=0) = 0.0281$

ii) $P(X>3) = 1 - P(X \le 3)$

$= 1 - 0.5981$

$= 0.4019$

7 A game requires 15 identical ordinary dice to be thrown in each turn.

Assuming the dice to be fair, find the following probabilities for any given turn.

(i) No sixes are thrown. [2]

(ii) Exactly four sixes are thrown. [3]

(iii) More than three sixes are thrown. [2]

Let x be number of 6s thrown

$$X \sim B(\overset{n}{15}, \overset{p}{\tfrac{1}{6}})$$

(i) $P(X=0) = \left(\frac{5}{6}\right)^{15} = 0.0649$

ii) $P(X=4) = {}^{15}C_4 \times \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{11}$

$= 0.1418$

iii) $P(X>3) = 1 - P(X \leq 3)$

$= 1 - 0.7685$

$= 0.2315$