Small Angles

Aside

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$$

For small angles

$$\sin x \approx x$$

$$\cos x \approx 1 - \frac{x^2}{2}$$

$$\tan x \approx x$$

where x is small enough for powers of x above x^2 to be neglected.

Degrees θ	Radians θ	$\sin\theta$	$\tan\theta$
2°	0.0349	0.0349	0.0349
10°	0.1745	0.1736	0.1763
20°	0.3491	0.3420	0.3640

Exercise 5F

1 When θ is small, find the approximate values of:

a $\dfrac{\sin 4\theta - \tan 2\theta}{3\theta}$ **b** $\dfrac{1-\cos 2\theta}{\tan 2\theta \sin\theta}$ **c** $\dfrac{3\tan\theta - \theta}{\sin 2\theta}$

2 When θ is small, show that:

a $\dfrac{\sin 3\theta}{\theta \sin 4\theta} = \dfrac{3}{4\theta}$ **b** $\dfrac{\cos\theta - 1}{\tan 2\theta} = -\dfrac{\theta}{4}$ **c** $\dfrac{\tan 4\theta + \theta^2}{3\theta - \sin 2\theta} = 4 + \theta$

3 **a** Find $\cos(0.244\text{ rad})$ correct to 6 decimal places.

b Use the approximation for $\cos\theta$ to find an approximate value for $\cos(0.244\text{ rad})$.

c Calculate the percentage error in your approximation.

d Calculate the percentage error in the approximation for $\cos 0.75$ rad.

e Explain the difference between your answers to parts **c** and **d**.

(P) **4** The percentage error for $\sin\theta$ for a given value of θ is 1%. Show that $100\theta = 101\sin\theta$.

(E/P) **5** **a** When θ is small, show that the expression $\dfrac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta}$ can be written as $9\theta + 2$. (3 mark

b Hence write down the value of $\dfrac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta}$ when θ is small. (1 mar

1 c $\dfrac{3\tan\theta - \theta}{\sin 2\theta}$ $\approx \dfrac{3\theta - \theta}{2\theta} = \dfrac{2\theta}{2\theta} = 1$

2 c $\dfrac{\tan 4\theta + \theta^2}{3\theta - \sin 2\theta} = 4 + \theta$ $\approx \dfrac{4\theta + \theta^2}{3\theta - 2\theta} = \dfrac{\theta(4+\theta)}{\theta}$

$= 4 + \theta$

1 a $\dfrac{\sin 4\theta - \tan 2\theta}{3\theta}$ $\approx \dfrac{4\theta - 2\theta}{3\theta} = \dfrac{2\theta}{3\theta} = \dfrac{2}{3}$

1 b $\dfrac{1 - \cos 2\theta}{\tan 2\theta \sin\theta}$ $\approx \dfrac{1 - \left(1 - \frac{(2\theta)^2}{2}\right)}{2\theta \times \theta} = \dfrac{2\theta^2}{2\theta^2} = 1$

2 a $\dfrac{\sin 3\theta}{\theta \sin 4\theta} = \dfrac{3}{4\theta}$

$\dfrac{\sin 3\theta}{\theta \sin 4\theta} \approx \dfrac{3\theta}{\theta \times 4\theta} = \dfrac{3\theta}{4\theta^2} = \dfrac{3}{4\theta}$

2 **b** $\dfrac{\cos\theta - 1}{\tan 2\theta} = -\dfrac{\theta}{4}$

$\dfrac{\cos\theta - 1}{\tan 2\theta} \approx \dfrac{1 - \frac{\theta^2}{2} - 1}{2\theta} = \dfrac{-\frac{\theta^2}{2}}{2\theta} = -\dfrac{\theta}{4}$

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3 **a** Find cos (0.244 rad) correct to 6 decimal places.

b Use the approximation for $\cos\theta$ to find an approximate value for cos (0.244 rad).

c Calculate the percentage error in your approximation.

d Calculate the percentage error in the approximation for cos 0.75 rad.

e Explain the difference between your answers to parts **c** and **d**.

a) $\cos 0.244 = 0.970379$

b) $1 - \dfrac{0.244^2}{2} = 0.970232$

$\%\ \text{error} = \dfrac{0.970232 - 0.970379}{0.970379} \times 100\ \%$

$= -0.015\ \%$

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c) $\cos 0.75 = 0.731689$

$1 - \dfrac{0.75^2}{2} = 0.71875$

% error $= \dfrac{0.71875 - 0.731689}{0.731689} \times 100$ %

$= -1.768$ %

d) The larger the value of Q the less accurate the approximation