Exercise 6B

1 Sketch, in the interval $-540° \leqslant \theta \leqslant 540°$, the graphs of:

a $y = \sec\theta$ b $y = \operatorname{cosec}\theta$ c $y = \cot\theta$

2 a Sketch, on the same set of axes, in the interval $-\pi \leqslant x \leqslant \pi$, the graphs of $y = \cot x$ and $y = -x$.

b Deduce the number of solutions of the equation $\cot x + x = 0$ in the interval $-\pi \leqslant x \leqslant \pi$.

3 a Sketch, on the same set of axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \sec\theta$ and $y = -\cos\theta$.

b Explain how your graphs show that $\sec\theta = -\cos\theta$ has no solutions.

4 a Sketch, on the same set of axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \cot\theta$ and $y = \sin 2\theta$.

b Deduce the number of solutions of the equation $\cot\theta = \sin 2\theta$ in the interval $0 \leqslant \theta \leqslant 360°$.

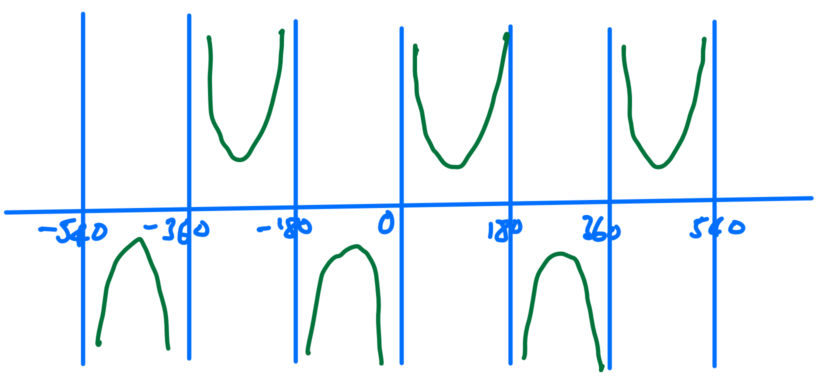
5 a Sketch on separate axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \tan\theta$ and $y = \cot(\theta + 90°)$.

b Hence, state a relationship between $\tan\theta$ and $\cot(\theta + 90°)$.

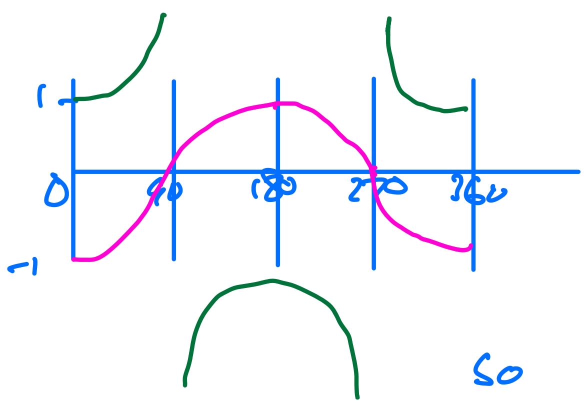
1 b) $-540 \leq \theta \leq 540$

$y = \operatorname{cosec}\theta$

$y = \frac{1}{\sin\theta}$



3)



$y = -\cos\theta$

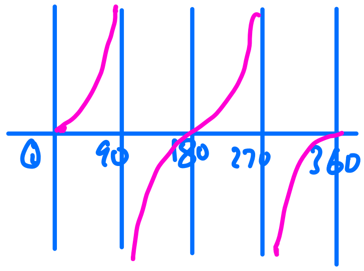
$y = \sec\theta$

no intersection

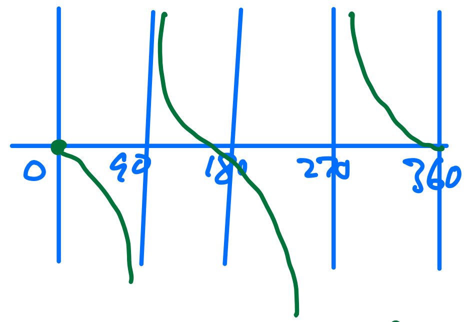
so $\sec\theta = -\cos\theta$ no solutions

5 a Sketch on separate axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \tan\theta$ and $y = \cot(\theta + 90°)$.

b Hence, state a relationship between $\tan\theta$ and $\cot(\theta + 90°)$.



$y = \tan\theta$



$y = \cot(\theta + 90)$

$\cot(\theta + 90°) = -\tan\theta$

Ⓟ **6 a** Describe the relationships between the graphs of:

i $y = \tan\left(\theta + \frac{\pi}{2}\right)$ and $y = \tan\theta$ **ii** $y = \cot(-\theta)$ and $y = \cot\theta$

iii $y = \operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ and $y = \operatorname{cosec}\theta$ **iv** $y = \sec\left(\theta - \frac{\pi}{4}\right)$ and $y = \sec\theta$

b By considering the graphs of $y = \tan\left(\theta + \frac{\pi}{2}\right)$, $y = \cot(-\theta)$, $y = \operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ and $y = \sec(\theta -$ state which pairs of functions are equal.

Ⓟ **7** Sketch on separate axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of:

a $y = \sec 2\theta$ **b** $y = -\operatorname{cosec}\theta$ **c** $y = 1 + \sec\theta$

d $y = \operatorname{cosec}(\theta - 30°)$ **e** $y = 2\sec(\theta - 60°)$ **f** $y = \operatorname{cosec}(2\theta + 60°)$

g $y = -\cot(2\theta)$ **h** $y = 1 - 2\sec\theta$

In each case show the coordinates of any maximum and minimum points, and of any points : which the curve meets the axes.

8 Write down the periods of the following functions. Give your answers in terms of π.

a $\sec 3\theta$ **b** $\operatorname{cosec}\frac{1}{2}\theta$ **c** $2\cot\theta$ **d** $\sec(-\theta)$

E/P **9 a** Sketch, in the interval $-2\pi \leqslant x \leqslant 2\pi$, the graph of $y = 3 + 5\operatorname{cosec} x$. **(3 mark**

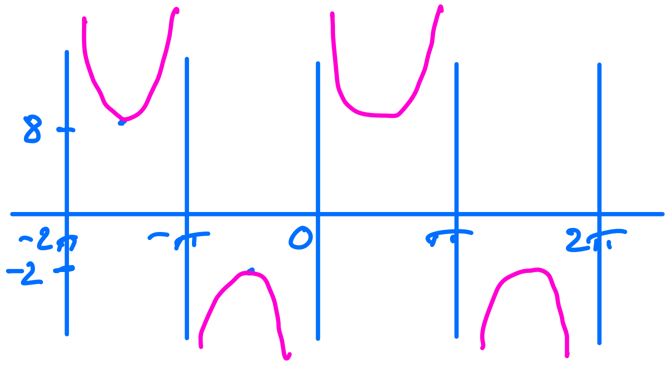
b Hence deduce the range of values of k for which the equation $3 + 5\operatorname{cosec} x = k$ has no solutions. **(2 mark**

E/P **10 a** Sketch the graph of $y = 1 + 2\sec\theta$ in the interval $-\pi \leqslant \theta \leqslant 2\pi$. **(3 mark**

b Write down the θ-coordinates of points at which the gradient is zero. **(2 mark**

c Deduce the maximum and minimum values of $\frac{1}{1 + 2\sec\theta}$, and give the smallest positive values of θ at which they occur. **(4 mark**

a)



no solutions for

$3+5\operatorname{cosec} x = k$

for $-2 < k < 8$

Exercise 6C

1 Rewrite the following as powers of $\sec\theta$, $\operatorname{cosec}\theta$ or $\cot\theta$.

a $\dfrac{1}{\sin^3\theta}$ b $\dfrac{4}{\tan^6\theta}$ c $\dfrac{1}{2\cos^2\theta}$ d $\dfrac{1-\sin^2\theta}{\sin^2\theta}$

e $\dfrac{\sec\theta}{\cos^4\theta}$ f $\sqrt{\operatorname{cosec}^3\theta\cot\theta\sec\theta}$ g $\dfrac{2}{\sqrt{\tan\theta}}$ h $\dfrac{\operatorname{cosec}^2\theta\tan^2\theta}{\cos\theta}$

2 Write down the value(s) of $\cot x$ in each of the following equations.

a $5\sin x = 4\cos x$ b $\tan x = -2$ c $3\dfrac{\sin x}{\cos x} = \dfrac{\cos x}{\sin x}$

3 Using the definitions of **sec**, **cosec**, **cot** and **tan** simplify the following expressions.

a $\sin\theta\cot\theta$ b $\tan\theta\cot\theta$

c $\tan 2\theta\operatorname{cosec} 2\theta$ d $\cos\theta\sin\theta(\cot\theta+\tan\theta)$

e $\sin^3 x\operatorname{cosec} x + \cos^3 x\sec x$ f $\sec A - \sec A\sin^2 A$

g $\sec^2 x\cos^5 x + \cot x\operatorname{cosec} x\sin^4 x$

(P) 4 Prove that:

a $\cos\theta + \sin\theta\tan\theta \equiv \sec\theta$ b $\cot\theta + \tan\theta \equiv \operatorname{cosec}\theta\sec\theta$

c $\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$ d $(1-\cos x)(1+\sec x) \equiv \sin x\tan x$

e $\dfrac{\cos x}{1-\sin x} + \dfrac{1-\sin x}{\cos x} \equiv 2\sec x$ f $\dfrac{\cos\theta}{1+\cot\theta} \equiv \dfrac{\sin\theta}{1+\tan\theta}$

1d) $\dfrac{1-\sin^2\theta}{\sin^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta} = \left(\dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \cot^2\theta$

1.f) $\sqrt{\operatorname{cosec}^3\theta \cot\theta \sec\theta}$

$= \sqrt{\frac{1}{\sin^3\theta} \times \frac{\cancel{\cos\theta}}{\sin\theta} \times \frac{1}{\cancel{\cos\theta}}}$

$= \sqrt{\frac{1}{\sin^4\theta}} = \sqrt{\operatorname{cosec}^4\theta} = \operatorname{cosec}^2\theta$

2c) $3\frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$

$3\tan x = \cot x$

$\frac{3}{\cot x} = \cot x$

$3 = \cot^2 x$

$\pm\sqrt{3} = \cot x$

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3 Using the definitions of **sec**, **cosec**, **cot** and **tan** simplify the following expressions.

a $\sin\theta \cot\theta$

b $\tan\theta \cot\theta$

c $\tan 2\theta \operatorname{cosec} 2\theta$

d $\cos\theta \sin\theta (\cot\theta + \tan\theta)$

e $\sin^3 x \operatorname{cosec} x + \cos^3 x \sec x$

f $\sec A - \sec A \sin^2 A$

g $\sec^2 x \cos^5 x + \cot x \operatorname{cosec} x \sin^4 x$

3b) $\tan\theta \cot\theta = \tan\theta \times \frac{1}{\tan\theta} = 1$

3d) $\cos\theta \sin\theta (\cot\theta + \tan\theta)$

$\cos\theta \sin\theta \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \right)$

$\cos\theta \sin\theta \left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos\theta} \right)$

$= \cos\theta \sin\theta \left(\frac{1}{\sin\theta \cos\theta} \right) = 1$

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g $\sec^2 x \cos^5 x + \cot x \operatorname{cosec} x \sin^4 x$

$= \frac{1}{\cos^2 x} \cos^5 x + \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \times \sin^4 x$

$= \cos^3 x + \cos x \sin^2 x$

$= \cos x (\cos^2 x + \sin^2 x)$

$= \cos x$

Classwork and Homework

Complete the rest of Exercise 6B

and Exercise 6C Q 1, 2, 3